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Estimation of a structural model of competition in the Colombian electricity spot market

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November 28, 2016

Contents

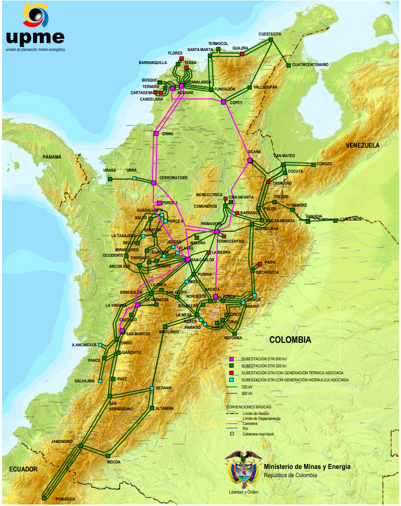
The Colombian electricity market

- Identify marginal costs of production as well as start-up costs and water opportunity costs for both thermal and hydro generators.
- Evaluate whether the current dispatch model improves efficiency in the Colombian electricity market compared with a traditional multi-unit auction setting.

The Colombian electricity market

- The Colombian electricity industry is very complex (Carranza, Riascos, Morán, & Bermeo, in press).
- There are 4 main activities: generation, transmission, distribution and retailing.
- The wholesale electricity market (MEM) was established in 1994 when generation and retailing were deregulated.
- This is a centralized market, interconnected through a national-wide network called *Sistema Interconectado Nactional* (SIN).
- The MEM consists of two separated markets: the forward (bilateral contract) market and the spot market.
- All production decisions are centralized by the *Centro Nacional de Despacho* (CND) and defined in the spot market.

Sistema Interconectado Nactional, 2013

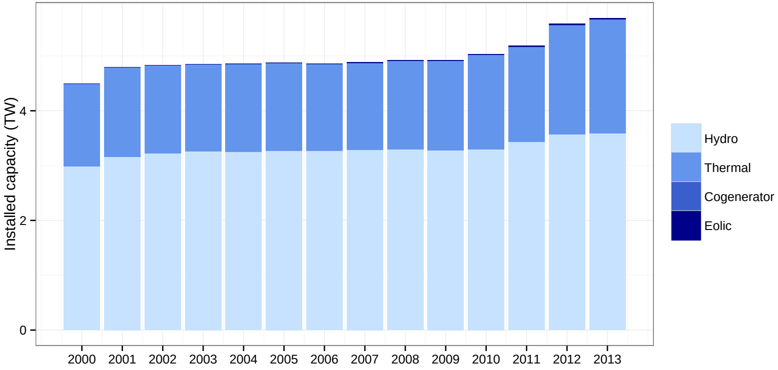


Source: UPME

Productive structure

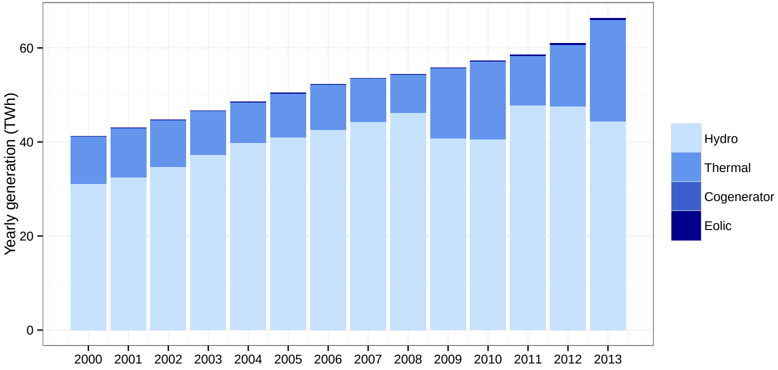
- Generation technology is primarily hydroelectric (hydro) and themoelectric (thermal).
- More than 63% of the total installed capacity is hydro.
- Few plants own most of the installed capacity: about 34% of all generators own 96% of total capacity.
- This productive structure is dominated by three large companies: *Emgesa*, *Empresas Públicas de Medellín* (EPM) and *Isagen*.
- These firms owned more than 56% of SIN's installed net capacity and almost 70% of the total water storage capacity.

Installed capacity



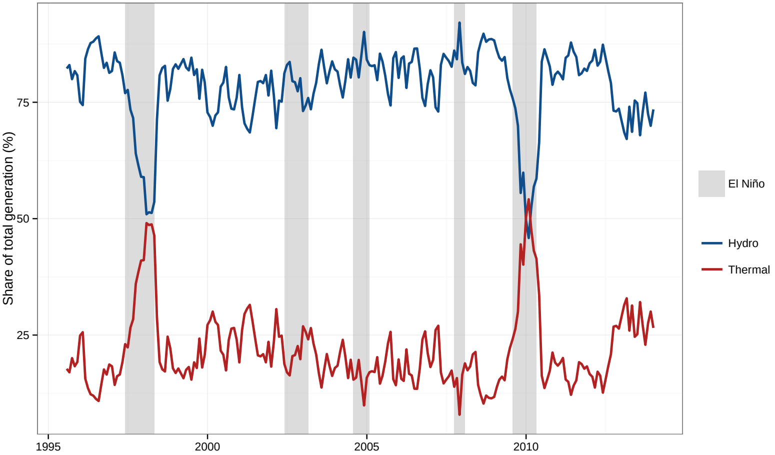
Source: Author's calculations based on data from XM

Annual generation



Source: Author's calculations based on data from XM

Share of monthly generation



Source: Author's calculations based on data from XM and IRI

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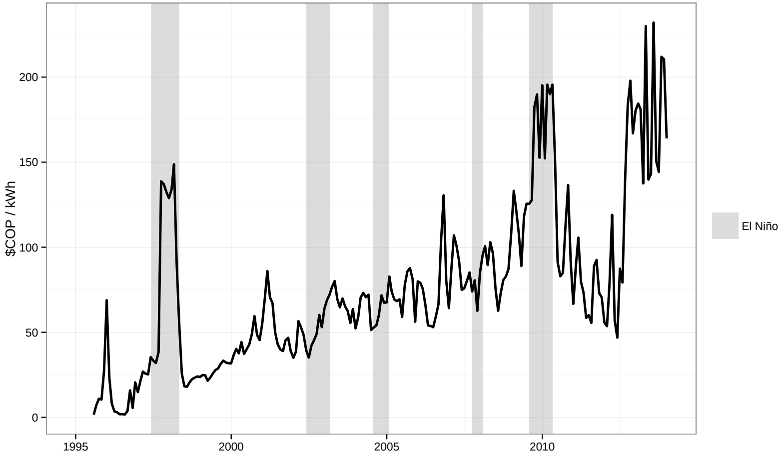
Monthly average spot price



Source: Author's calculations based on data from XM and IRI

The spot market

All the electricity in Colombia is allocated in the spot market. The mechanism takes the form of an augmented set of uniform-price multi-unit auctions.

- Firms submit supply schedules to satisfy demand in an hourly basis.
- The bidding structure and the definition of the market price (spot price) differ across three periods:
  1. 1995–2001: Bids consisted on a daily schedule of 24 hourly prices and available capacity.
  2. 2001–2009: Bids consisted on a unique daily price and a schedule of 24 available capacity levels.
  3. 2009–: Bids for thermal units also include start-up and shutdown costs parameters. (complex bids)

  This study analyzes firms' behavior after 2009.

Auction mechanism

Economic dispatch (the day before production):

1. Firms offer electricity for each hour of the next day (A set of 24 simultaneous multi-unit auctions).
2. The CND uses bids to compute a minimum cost daily generation schedule based on a demand forecast, as well as accounting for a set of multiple technical and network restrictions.

Auction mechanism

Real dispatch (the day of production):

1. Generators produce electricity according to the economic dispatch.
2. Deviations from the economic dispatch are centrally solved by the market operator.

Auction mechanism

Ideal dispatch (the day after production):

1. The CND uses bids to compute an minimum cost daily generation schedule based on the realized demand, units' observed availability, and accounting a set of multiple technical restrictions but ignoring network restrictions.
2. The hourly market prices is computed as the price bid of the marginal bidder that is not considered to be inflexible for the respective hourly period.
3. If, given the hourly prices, there are dispatched units with negative earnings, an uplift to the price is computed in order to ensure that generation by merit makes non-negative profits.

The model

The model is based on works by Reguant (2014) and Camelo, de Castro, Papavasiliou, Riascos, and Oren (2016).

- Consider $i = 1, \ldots, N$ firms that compete to produce and sell electricity in the MEM.
- Each firm owns a set of $j = 1, \ldots, J_i$ generating units.
- The hourly demand for energy is defined as $\tilde{D}_h = D_h + \varepsilon_h$, where ε_h is *ex ante* unknown.
- The process that generates ε_h, $F(\varepsilon_h)$, is known to all firms.

Bidding structure

- Every quarter of the year, firms are requested to submit a vector of monetary start-up costs bids $\mathbf{c}_i = \{A_{ij}\}$.
- For each day, within a quarter, firms are requested to submit simple bids $\mathbf{b}_i = \{b_{ij}, g_{ijh}\}$ conditional on $\mathbf{c}_i$, where
  - b_{ij} is a bid price (constant across the 24 ours of the day).
  - g_{ijh} is the production capacity available for hour h.

Ideal dispatch

All submitted bids are collected by the market operator to define the ideal dispatch and the market price by solving:

$$\min_{\{q_{ijh}\}} \quad \sum_{h=0}^{23} \sum_{i=1}^{N} \sum_{j=1}^{J_i} b_{ij} q_{ijh} + A_{ij} \mathbf{1}_{ijh}^{\text{start}} \tag{1a}$$

subject to

$$\left\{ \sum_{i=1}^{N} \sum_{j=1}^{J_i} q_{ijh} - \tilde{D}_h \right\} \geq \mathbf{0} \tag{1b}$$

$$\left\{ \mathbf{k}_{ijh}^{1}(q_{ijh}, s_{ijh}, \mathbf{r}_{ijh}) \right\} = \mathbf{0} \tag{1c}$$

$$\left\{ \mathbf{k}_{ijh}^{2}(q_{ijh}, s_{ijh}, \mathbf{r}_{ijh}) \right\} \geq \mathbf{0} \tag{1d}$$

Ideal dispatch

where

- q_{ijh} is the actual generation of unit j at hour h
- s_{ijh} equals 1 if unit j is switched on at h, and 0 otherwise
- $\mathbf{r}_{ijh}$ is a vector of unit j's technical parameters
- $\mathbf{k}^1_{ijh}(\cdot)$ and $\mathbf{k}^2_{ijh}(\cdot)$ are non-linear vector functions. (See Camelo et al. (2016))

Ideal dispatch

- Equation (1b) represents the market clearing condition, which is a usual restriction in most energy auctions.
- Equations (1c) and (1d) are the most particular characteristic of the Colombian setting. They represent the set of technical restrictions that need to be satisfied in order for the dispatch to be feasible.
- Given the equilibrium dispatch the market clearing price p_h is computed as the marginal price among all dispatched bids, as in a uniform multi-unit auction.

Profits of the firm

- All dispatched units are paid p_h for each kWh produced at the respective hour of the day.
- Additionally, every thermal unit j for which

$$\sum_{h=1}^{23} p_h q_{ijh} < \sum_{h=1}^{23} b_{ij} q_{ijh} - A_{ij} \mathbf{1}_{ijh}^{\{\text{start}\}} \tag{2}$$

is also paid with an uplift to the hourly price, denoted by ΔI, which depends on the market outcomes.

Information set

- At the time of bidding firms are still uncertain about other firms' strategies as well as the realization of ε_h.
- Therefore, firm i will choose a bidding strategy in order to maximize its expected profits, conditional on a given distribution of other firms' bids as well as on a set of public and private information.
- Public information common to all firms includes: demand forecasts, dams' water storage levels and inflows, fossil fuel prices as well as the technical parameters of all generating units.
- Firm's private values may consist on: maintenance strategy or unit unavailabilities and bilateral contracts.

Profits of the firm

- Given the available information set, firm i's expectations are taken over its own beliefs about other firms' strategies.
- Denoting S as the set of all feasible combinations of units being dispatched, the expected profits of firm i for a given day can be expressed as

$$E_{-i}[\Pi_i(\mathbf{b}, \mathbf{c})] = \sum_{s \in S} \Pr(s \mid \mathbf{b}_i, \mathbf{c}_i) \, E_{-i}[\Pi_i(\mathbf{b}_s, \mathbf{c}_s) \mid s] \,, \qquad (3)$$

where $\Pr(s \mid \mathbf{b}_i, \mathbf{c}_i)$ defines the probability that a combination of units s is dispatched, conditional on firm i's own bids.

Profits of the firm

- Conditional on s, the market outcomes are only determined by the set of bids that are dispatched, denoted by $\{\mathbf{b}_s, \mathbf{c}_s\}$.
- Firm i's profit function at a given state s and bid strategies $\{\mathbf{b}_s, \mathbf{c}_s\}$ is given by

$$\Pi_i(\mathbf{b}_s, \mathbf{c}_s) = \left[\sum_{h=0}^{23} (Q_{ih}(\mathbf{b}_s, \mathbf{c}_s) - v_{ih})\, p_h(\mathbf{b}_s, \mathbf{c}_s) - \Delta I(\mathbf{b}_s, \mathbf{c}_s) v_{ih} \right.$$
$$\left. + \Delta I(\mathbf{b}_s, \mathbf{c}_s) \sum_{j=1}^{J_i} \mathcal{I}_{ij}(\mathbf{b}_s, \mathbf{c}_s) q_{ijh}(\mathbf{b}_s, \mathbf{c}_s) \right] \qquad (4)$$
$$- \sum_{j=1}^{J_i} C_{ij}(\mathbf{q}_{ij}(\mathbf{b}_s, \mathbf{c}_s)),$$

Profits of the firm

where

- $Q_{ih}(\cdot)$ is the total quantity produced by firm i at hour h
- v_{ih} is the firm's aggregate net sales position in the market of bilateral contracts.
- $\mathcal{I}_{ij}$ is an indicator function defined as follows

$$\mathcal{I}_{ij} = \begin{cases} 1, & \text{if } j \text{ is thermal and } \sum_{h=1}^{23} p_h q_{ijh} < \sum_{h=1}^{23} b_{ij} q_{ijh} - A_{ij} \mathbf{1}_{ijh}^{\{\text{start}\}} \\ 0, & \text{otherwise} \end{cases}$$

- $C_{ij}(\cdot)$ represents the total daily costs function of unit j, which depends on the vector of hourly equilibrium unit quantities.
- We assume that firms' dynamic incentives are summarized in their cost structures.

Costs structure for thermal units

- The dynamic problem of thermal units arises due to the existence of ramping and start-up costs.
- They represent the inflexibility of a thermal unit to rapidly change its production levels.
- We use the costs specification proposed by (Reguant, 2014):

$$C_{ij}(\mathbf{q}_{ij}) = \sum_{h=0}^{23} \gamma_{ij1} q_{ijh} + \frac{\gamma_{ij1}}{2} \tilde{q}_{ijh}^2 + \frac{\gamma_{ij3}}{2} (q_{ijh} - q_{ijh-1})^2 + \alpha_{ij} \mathbf{1}_{ijh}^{\{\text{start}\}} \quad (5)$$

where

- γ_{ij1} and γ_{ij2} represent j's marginal costs of production
- γ_{ij3} represents the ramping costs,
- $\tilde{q}_{ijh} = \max\{q_{ijh} - \underline{q}_{ij}, 0\}$ is the unit's production over its minimum
- α_{ij} is the total cost incurred whenever j gets switched on.

Costs structure for hydro units

- The dynamic problem of hydro units arises because their capacity to store energy in the form of water.
- This implies an intertemporal opportunity cost defined as the value of future payoffs the firm gives up in order to produce energy (by releasing the water) in the current period.
- We follow the characterization proposed by Carranza, Balat, and Martin (2015)

$$C_{ij}(\mathbf{q}_{ij}) = \left(\sum_{h=0}^{23} \lambda_{ij} q_{ijh} \right) + \Psi_{ij}(\mathbf{q}_{ij}, \boldsymbol{\omega}_{ij}), \qquad (6)$$

where

- λ_{ij} is the marginal costs of production
- $\Psi_{ij}(\cdot)$ represents firm i's valuation for the sum of its future expected profits associated with unit j
- $\boldsymbol{\omega}_{ij}$ is the current state of water storage and inflows levels.

Equilibrium and optimality conditions

- The equilibrium of the model can be expressed as the solution to a 2-stages sequential game:
  - **Stage 1**: Each firm *i* chooses the values of $\mathbf{c}_i$ which will be committed during the following 90 days
  - **Stage 2**: Conditional on her action upon $\mathbf{c}_i$, the firm chooses a supply schedule $\mathbf{b}_i$ to its maximize profits for each daily auction
- We characterize the optimiality conditions for a Bayesian Perfect equilibrium using the backward induction solution concept.

Optimality conditions: Stage 2

- Given the Markovian structure of the cost functions, each sub-game of the second (competition) stage can be solved as a conditionally independent simultaneous auction.
- In each day within a given quarter of the year, firms will choose simple bid strategies as to maximize their expected daily profits:

$$\max_{\mathbf{b}_i} \sum_{s \in S} \Pr(s \mid \mathbf{b}, \mathbf{c})\, E_{-i}[\Pi_i((\mathbf{b}_i, \mathbf{b}_{-i}), \mathbf{c}) \mid s, \mathbf{c}_i] \,. \tag{7}$$

- We focus on the first-order conditions with respect to the price offers.
- This is the usual approach in the energy auctions literature (Hortacsu & Puller, 2008; Kastl, 2011; Reguant, 2014; Wolak, 2003).

Optimality conditions: Stage 2

Then optimal strategy for simple bidding must satisfy the following first-order condition:

$$\sum_{s\in S} \Pr(s \mid \mathbf{b},\mathbf{c}) \frac{\partial E_{-i}[\Pi(\mathbf{b},\mathbf{c}) \mid s, \mathbf{c}_i]}{\partial b_{ij}} + \sum_{s\in S} \frac{\partial \Pr(s \mid \mathbf{b},\mathbf{c})}{\partial b_{ij}} E_{-i}[\Pi(\mathbf{b},\mathbf{c}) \mid s, \mathbf{c}_i] = 0. \tag{8}$$

- The first term in can be interpreted in a similar fashion as in a usual multi-unit auction setup.
- The second term represents the extent to which small changes in b_{ij} affect the probability that any unit belonging to firm i will sell a positive quantity of electricity during the day.

Optimality conditions: Stage 2

- Notice that the second derivative term in (8) is non-zero only when b_{ij} or A_{ij} are high enough so that j is the most costly unit in s, and there is unit $l \notin s$, such that the alternative combination of units $\hat{s} = \{s_{-j}, l\}$ is technically feasible and that the cost of the resulting dispatch is sufficiently low.
- Therefore, we follow (Reguant, 2014) and assume that

$$\sum_{s \in S} \frac{\partial \Pr(s \mid \mathbf{b}, \mathbf{c})}{\partial b_{ij}} E_{-i}[\Pi(\mathbf{b}, \mathbf{c}) \mid s, \mathbf{c}_i] \approx 0. \tag{9}$$

- This assumption allows us to express the optimality conditions for simple bids in a similar manner as in usual uniform-price auction settings.

Optimality conditions: Stage 2

Conditional on $\mathbf{c}_i$, firm i's optimal strategy for simple bids must satisfy, for all $s \in S$ and $j \in \{1, \ldots, J_i\}$,

$$b_{ij} = \bar{\zeta}_{ij} - \frac{\sum_{h=0}^{23} E_{-i}\left[Q_{ih} - (1 + \frac{\partial \Delta I}{\partial b_{ij}}) v_{ih} \;\middle|\; s, p_h = b_{ij}\right]}{\sum_{h=0}^{23} E_{-i}\left[\frac{\partial Q_{ih}}{\partial b_{ij}} \;\middle|\; s, p_h = b_{ij}\right]} + \frac{\sum_{l=1}^{J_i} \sum_{h=0}^{23} E_{-i}\left[\frac{\partial \Delta I}{\partial b_{ij}} q_{ilh} + \frac{\partial q_{ilh}}{\partial b_{ij}} \Delta I \;\middle|\; s, p_h = b_{ij}, \mathcal{I}_{il} = 1\right] \phi_{il}}{\sum_{h=0}^{23} E_{-i}\left[\frac{\partial Q_{ih}}{\partial b_{ij}} \;\middle|\; s, p_h = b_{ij}\right]} + \frac{\sum_{l=1}^{J_i} \sum_{h=0}^{23} E_{-i}[\Delta I \times q_{ilh} \mid s, p_h = b_{ij}, \mathcal{I}_{il} = 1] \frac{\partial \phi_{il}}{\partial b_{ij}}}{\sum_{h=0}^{23} E_{-i}\left[\frac{\partial Q_{ih}}{\partial b_{ij}} \;\middle|\; s, p_h = b_{ij}\right]}, \quad (10)$$

Optimality conditions: Stage 2

where

- $\bar{\zeta}_{ij}$ represents a weighted average of the daily marginal cost:

$$\bar{\zeta}_{ij} = \frac{\sum_{h=0}^{23} E_{-i}\left[\frac{\partial C_{ij}}{\partial q_{ijh}}\left(\frac{\partial q_{ilh}}{\partial b_{ij}}\right) \;\middle|\; s, p_h = b_{ij}\right]}{\sum_{h=0}^{23} E_{-i}\left[\frac{\partial Q_{ih}}{\partial b_{ij}} \;\middle|\; s, p_h = b_{ij}\right]}. \tag{11}$$

- ϕ_{ij} defines the probability for unit j of being paid the price-uplift ΔI, conditional on the bid schedule and state variables:

$$\phi_{ij}(\mathbf{b}_i, \cdot) \equiv \Pr\left(\mathcal{I}_{ij} = 1 \mid \mathbf{b}_i, \cdot\right). \tag{12}$$

Optimality conditions: Stage 2

- The first line in equation (10) represents the usual definition of optimal bidding in a traditional uniform-price auction setting.
- The last two terms arise due to the presence of complex bids and the introduction of the uplift to the market price ΔI.

Optimality conditions: Stage 1

- In the first stage of the sequential game, firms are aware that their complex bids decisions will be committed for each of the 90 sub-games of the second stage.
- Therefore, firm i will choose a complex bid strategy, $\mathbf{c}_i$, so as to maximize the total sum of its expected profits during the following 90 days.
- Firm i to account for the fact that $\mathbf{c}_i$ will affect i's profits not only directly, but through its own simple bid strategy as well.

Optimality conditions: Stage 1

- Formally, let B and C denote the space of simple and complex bids, respectively. Also, let Ω be the state space.
- Define firm i's optimal strategy for simple bids at auction t as the vector function $\boldsymbol{\beta} : C \times \Omega \to B$ such that, for any given $\mathbf{c}_i \in C$ and $\omega_t \in \Omega$, $\mathbf{b}_{it} = \boldsymbol{\beta}_i(\mathbf{c}_i, \omega_t)$ satisfies the conditions implied by equation (10).
- Then, we can write firm i's optimization problem at the first stage of the game as follows:

$$\max_{\mathbf{c}_i} \; E_{-i}\left[\sum_{t=1}^{90} \Pi_i(\mathbf{b}_t, \mathbf{c})\right], \quad \text{s.t. } \mathbf{b}_{it} = \boldsymbol{\beta}_i(\mathbf{c}_i, \omega_t). \qquad (13)$$

Optimality conditions: Stage 1

Consequently, the first-order necessary conditions for this optimization problem are given by,

$$\sum_{t=1}^{90}\sum_{s\in S}\Pr\left(s \mid \boldsymbol{\beta}_{it}(\mathbf{c}_i),\mathbf{c}\right)\frac{\partial E_{-i}[\Pi_i(\boldsymbol{\beta}_{it}(\mathbf{c}_i),\mathbf{c}) \mid s]}{\partial A_{ij}}+ \sum_{t=1}^{90}\sum_{s\in S}\frac{\partial \Pr\left(s \mid \boldsymbol{\beta}_{it}(\mathbf{c}_i),\mathbf{c}\right)}{\partial A_{ij}}E_{-i}[\Pi_i(\boldsymbol{\beta}_{it}(\mathbf{c}_i),\mathbf{c}) \mid s]=0. \tag{14}$$

Notice that, complex bids affect firm i's daily profits through both probability of having any of its unit dispatched and through the definition of prices and quantities.

Estimation

- We use observed bidding data to recover marginal as well as start-up and water opportunity costs for thermal and hydro units.
- In particular, we estimate the structural parameters of the cost function defined in equation (5) for every firm i:

$$\boldsymbol{\theta}_i = \{\alpha_i, \gamma_i, \lambda_i\} . \tag{15}$$

- We use the empirical moments implied by the optimality conditions of the bidding game defined by equations (10) and (14).

Estimation

- Parameters are estimated by the Generalized Method of Moments (GMM).
- The first step is to estimate marginal costs and water opportunity costs from equation (10).
- Then, given estimates for $\{\gamma_i, \lambda_i\}$, we proceed to estimate the start-up cost parameters from equation (14).
- To estimate the firm's beliefs we adapt the bootstrapping procedure standard in the auction literature (Hortacsu & McAdams, 2010; Kastl, 2011; Reguant, 2014).
- To separately estimate marginal production costs and water opportunity costs for hydro generators we adapt the identification strategy proposed by Martin (2015).
- The derivative terms are approximated using the smoothing method proposed by Wolak (2007).

The bootstrapping algorithm

The bootstrapping algorithm we employ for this study can be summarized as follows:

1. Fix bidder i's strategies in auction t
2. Randomly draw strategies of other firms $k \neq i$ from a sample of N similar days, conditioning on a set of observed state variables
3. Compute the market equilibrium using the computational algorithm proposed by Camelo et al. (2016)
4. Repeat steps 2-3 M times to obtain a distribution of market outcomes

The bootstrapping algorithm

There are two aspects that make our bootstrap simulation procedure similar to the one used by Reguant (2014) and different from other applications.

1. The market clearing is defined as the solution to a complex optimization problem and cannot be necessarily replicated through a standard uniform-price multi-unit auction.
2. As in the model introduced by Reguant (2014), firms also face uncertainty over their own equilibrium supply curve as the set of units that will be dispatched is also random due to the presence of complex bids.

The bootstrapping algorithm

- Our model differs form the one of Reguant (2014) mainly in the fact that complex bids are submitted on a quarterly basis.
- This requires to compute an estimate of the firms' expected sequence of their profits for the following 90 days.
- Because the available data for complex bids is short, we cannot directly estimate the underlying joint distribution of this sequence.
- We use same estimates of the expectation terms used to construct the empirical analogue of optimality conditions for simple bids.
- The underlying assumption is that, on average, firms' predictions about the evolution of the state variables are accurate enough, so that their expectations when submitting a complex bid are the same as in the immediate moment before submitting their simple bid strategies.

Approximation of derivatives

The approximate versions for the derivative terms of firm i's residual demand and aggregate supply used to construct the empirical moments are the following:

$$\widehat{\frac{\partial D^{R,bs}_{iht}}{\partial b_{ijt}}} = \frac{1}{\nu} \sum_{k \neq i} \sum_{(k,j) \in s^{bs}} g_{kjht} \mathcal{K}\left(\frac{b_{ljt} - p^{bs}_{ht}}{\nu}\right) \tag{16}$$

$$\widehat{\frac{\partial Q^{bs}_{iht}}{\partial b_{ijt}}} = \frac{1}{\nu} \sum_{(i,j) \in s^{bs}} g_{ijht} \mathcal{K}\left(\frac{b_{ijt} - p^{bs}_{ht}}{\nu}\right) \tag{17}$$

where bs denotes a bootstrap sample, $\mathcal{K}$ is a Kernel density weight and ν is a bandwith parameter.

Moment conditions

The empirical moment conditions implied by equation (10) is given by

$$m_{ijt}(\theta_i, \nu, M) =$$

$$\frac{1}{M}\sum_{bs=1}^{M}\sum_{h=0}^{23}\mathbf{1}\{j \text{ in}\}\left[\widehat{\frac{\partial p_{ht}^{bs}}{\partial b_{ijt}}}\left(\left(b_{ijt} - \bar{\zeta}_{ijt}(\theta_i)\right)\widehat{\frac{\partial D_{iht}^{R,bs}}{\partial b_{ijt}}} + Q_{ih}^{bs} - \left(1 + \widehat{\frac{\partial \Delta I_t^{bs}}{\partial b_{ijt}}}\right)v_{ih}\right) + \right.$$

$$\left.\sum_{l=1}^{J_i}\mathbf{1}\left\{\mathcal{I}_{ilt}^{bs} = 1\right\}\left(\left(\widehat{\frac{\partial \Delta I_t^{bf}}{\partial b_{ijt}}} q_{ilht}^{bs} + \widehat{\frac{\partial q_{ilht}^{bs}}{\partial b_{ijt}}}\Delta I_t^{bs}\right)\hat{\phi}_{ilt}^{bs} + \left(\Delta I_t^{bs} \times q_{ilht}^{bs}\right)\widehat{\frac{\partial \phi_{ilt}^{bs}}{\partial b_{ijt}}}\right)\right]$$

where the empirical versions of the marginal cost functions for thermal and hydro units are given by

$$\bar{\zeta}_{ijt}(\gamma_i) = \gamma_{ij1} + \gamma_{ij2}\tilde{q}_{ijh} + \gamma_{ij3}(2q_{ijh} - q_{ijh-1} - q_{ijh+1}) + \epsilon_{ijt},$$
$$\bar{\zeta}_{ijt}(\lambda_i) = \lambda_{ij} + \psi(\mathbf{q}_{ijt}, \boldsymbol{\omega}_{ijt}) + \varepsilon_{ijt},$$

and $\psi(\cdot)$ is a non-parametrical function (see Martin (2015)).

GMM Estimation

The GMM estimator of θ_i is given by

$$\theta_i^* = \arg\min_{\theta_i} \, [Z_t' m_{ijt}(\theta_i, \nu, M)]' \, \Phi [Z_t' m_{ijt}(\theta_i, \nu, M)] \tag{18}$$

where Z is a matrix of instruments assumed to be orthogonal to ϵ and ε. Finally, Φ is a weighting matrix.

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