

Motives and Consequences of Libor Misreporting

How Much Can We Learn From Banks' Misleading Submissions

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Libor (London Interbank Offered Rates):

- Are estimates of average unsecured funding rates in interbank markets
- Are computed based on daily quotes submitted by a panel of contributor banks
- Are the primary benchmark for short term ($<1\text{yr}$) interest rates globally
- Hundreds of trillions of dollars worth of financial contracts are pegged to Libor

Table 1

Reference rate estimated notional volumes and maturity concentrations

Rate	Currency Type	Notional o/s (\$TN)¹	Main Maturity Concentration
LIBOR	USD	\$150–160 TN	1 week, 1m and 3m and 6m
	GBP	\$30 TN	3m; then 1m & 6m
	JPY	\$30 TN	3m and 6m
	CHF	\$6.5 TN	3m and 6m
	EUR	\$2 TN	Low across all tenors
EURIBOR	EUR	\$150–180 TN	1m, 3m and 6m
TIBOR	JPY	\$5 TN	6m and 3m

Source: MPG Final Report currency Market Footprint overviews.

¹ Gross Volume

Background

How is Libor Calculated?

- Survey: “At what rate could you borrow funds, were you to do so by asking for and then accepting inter-bank offers in a reasonable market size just prior to 11 am?”
- The regulator computes a trimmed average of the banks' submissions after removing the 25% highest and 25% lowest.
- The regulator publishes the reference rate and the quotes submitted by all banks with their identities.
- In 2007 - 2010 there were 16 banks in the USD Libor panel (the largest).

- In recent years, several banks have been accused of attempts to manipulate Libor
- Some evidence suggest that during the financial crisis Libor rates do not accurately reflect funding costs
- Many banks in the USD Libor panel have payed penalties adding up to \$9bn (CFR - May 21, 2015)

Banks Incentives

Why misreport borrowing costs or try to manipulate the rate?

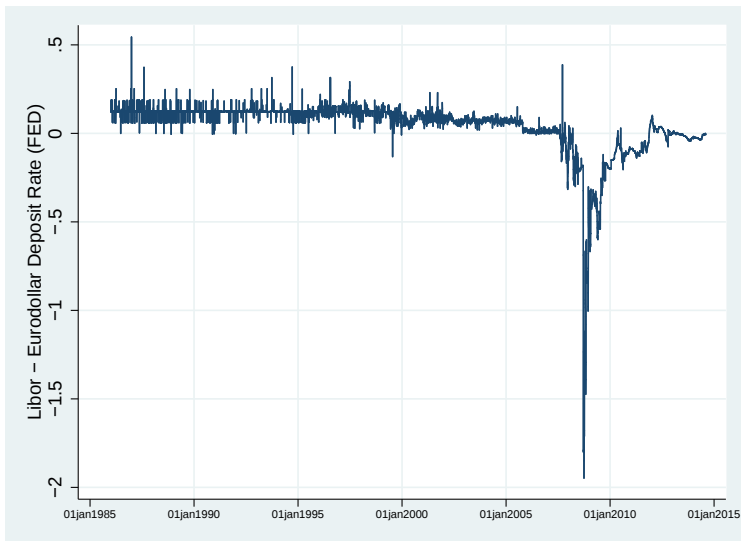
- Portfolio exposures:
 - Banks borrow and lend outside the interbank market at rates indexed to Libor
 - Mortgages, Commercial loans, Floating rate bank notes, etc.
 - Banks portfolio's returns depend on Libor through derivatives tied to this rates (Interest Rate Swaps).
- Signaling: Borrowing costs are signals of credit worthiness and liquidity

What the Paper Does

- Model the Libor quotes submission process as a Bayesian game: Snider & Youle (2012) , Chen (2013) and Youle (2014)
- Identify and estimate the model applying tools from the empirical auctions literature
- Identification from quotes alone in the presence of dependent (non i.i.d.) unobserved heterogeneity
 - Estimate a lower bound on the “truthful” Libor (absent any misreporting).
 - Recover parameters that determine banks’ incentives to misreport their costs
 - Recover the distributions of the borrowing costs
- To what extent misreporting was driven by portfolio exposures or signaling?
- Forthcoming: Analyze counterfactual scenarios to inform policy decisions on the design of interest rate benchmarks

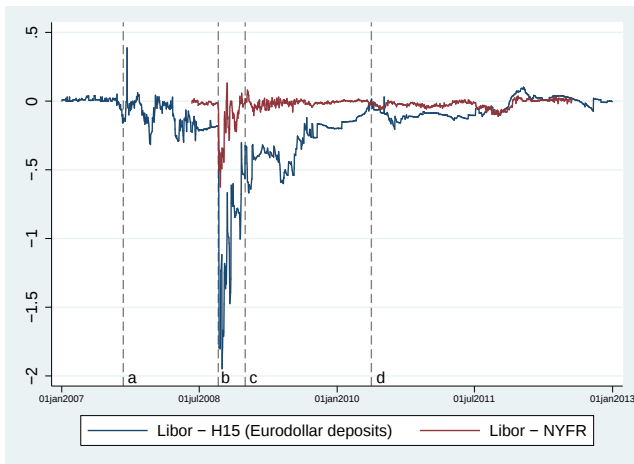
Empirical Evidence

Kuo, Skeie & Vickery (2012): 3M USD Libor vs Eurodollar Deposits Rate (FED)



Empirical Evidence

Kuo, Skeie & Vickery (2012): 3M USD Libor vs Eurodollar Deposits Rate (FED) & ICAP NYFR



A Model of Bank's Quotes

- There is a fixed number of banks in the panel: N
- Each day t , bank i observes its borrowing cost $s_{i,t}$ and submits a quote $r_{i,t}$
- The regulator computes the reference rate $\tilde{r}(r_t)$ (Libor) based on bank's quotes.
- $\tilde{r}_t = \frac{1}{N} \sum_{k=\underline{n}+1}^{\bar{n}-1} r_t^{(k)}$, where $r_t^{(1)} \leq r_t^{(2)} \leq \dots \leq r_t^{(N)}$ are the ordered quotes.
- Bank i chooses $r_{i,t} > 0$ to maximize

$$E \left[\underbrace{\alpha_i \tilde{r}(r_{i,t}, r_{-i,t})}_{\text{portfolio}} + \underbrace{v_i (\tilde{r}(r_{i,t}, r_{-i,t}) - r_{i,t})}_{\text{signaling}} - \underbrace{\gamma_i (s_{i,t} - r_{i,t})^2}_{\text{cost}} \middle| \mu_t \right]$$

- $v_i > 0, \gamma_i > 0$ and α_i are parameters determining individual preferences. All common knowledge.

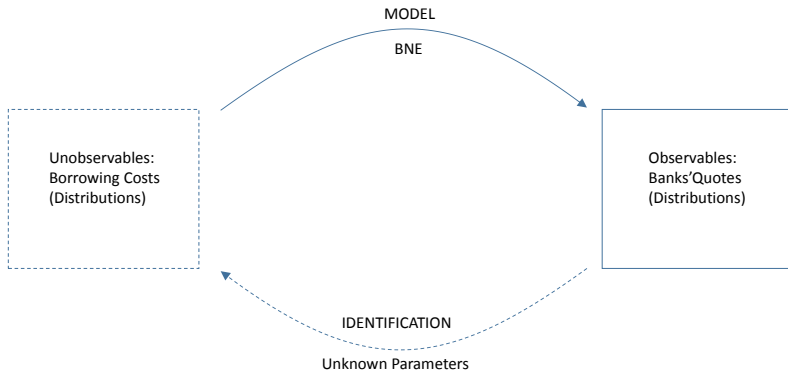
A Model of Bank's Costs

- Borrowing costs $s_{i,t}$ are determined by a common component μ_t and a private shock $\varepsilon_{i,t}$
- $s_{i,t} = \mu_t + \varepsilon_{i,t}$, $\varepsilon_{i,t} \stackrel{iid}{\sim} F_i$, $E[\varepsilon_{i,t}|\mu_t] = 0$
- F_i 's and μ_t are common knowledge
- F_i is absolutely continuous w.r.t. the Lebesgue measure
- For $i \neq j$, $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$ are independent
- For $i \neq j$, $s_{i,t}$ and $s_{j,t}$ are independent, conditional on μ_t

Bayesian Nash Equilibrium

- A set of strategies $\{\rho_i | \rho_i : \mathcal{S}_i \rightarrow \mathcal{R}_i\}_{i=1}^N$ such that, for all i , and all $s_i \in \mathcal{S}_i$, $r_i = \rho_i(s_i)$ maximizes $E[u_i(\cdot) | \mu_t]$ given all other banks' strategies ρ_{-i}
- There is a BNE in pure non-decreasing strategies (Athey, 2001)
- In any BNE, the strategies are strictly increasing in the support of s_i
- Any BNE satisfies a system of N necessary (first order) conditions
- For $i \neq j$, $r_{i,t} = \rho_i(s_{i,t})$ and $r_{j,t} = \rho_j(s_{j,t})$ are independent, conditional on μ_t
- Conjecture: Given μ_t , there is a unique BNE in pure strategies.

Structural Approach



- Let $r_t = (r_{1,t}, \dots, r_{N,t}) \sim G$ be a draw from the distribution of observable quotes (an observation)
- There is $s_t = (s_{1,t}, \dots, s_{N,t}) \sim F$, a draw from the distribution of costs such that: $r_t = \rho_1(s_{1,t}; \theta) = (\rho_1(s_{1,t}; \theta), \dots, \rho_N(s_{N,t}; \theta))$
- Then: $F(s) = G(\rho(s; \theta))$
- G is identified from multiple observations $\{r_t\}_{t=1}^T$
- $\rho(s; \theta)$ is known up to a vector of parameters θ
- Each θ determines a specific distribution F
- Given θ , $s_t = \rho^{-1}(r_t; \theta)$

- A necessary condition for ρ to be a vector of equilibrium strategies is that for all $i \in \{1, \dots, N\}$, and all $s_i \in S_i$

$$\beta_i \phi_i(\rho_i(s_i) | \mu) - v_i + (s_i - \rho_i(s_i)) = 0$$

- $r_i = \rho_i(s_i)$
- $\phi_i(r_i | \mu) = \frac{\partial E[\tilde{r}(r_i, r_{-i}) | \mu]}{\partial r_i}$
- $\phi_i(r_i | \mu)$ is the probability that r_i is included in the computation of $\tilde{r}$. [More](#)
- Wlog, $2\gamma_i = 1$ and $\beta_i = \frac{\alpha_i + v_i}{\tilde{N}}$

Inverse Equilibrium Strategies

- Inverse equilibrium strategies can be found without having to solve for the equilibrium: Guerre, Perrigne & Young (2000)
- If $r_{i,t} = \rho_i(s_{i,t}; \mu_t)$, then:

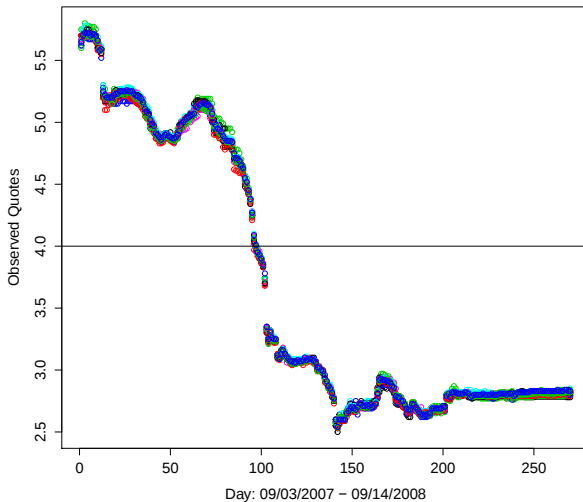
$$s_{i,t} = \rho_i^{-1}(r_{i,t}; \mu_t) = r_{i,t} - \beta_i \phi_i(r_{i,t} | \mu_t) + v_i$$

- Equivalently (in a panel fashion):

$$r_{i,t} = -v_i + \beta_i \phi_i(r_{i,t} | \mu_t) + \underbrace{\mu_t + \varepsilon_{i,t}}_{s_{i,t}}$$

- The ideal would be to estimate the parameters β_i , the “fixed effects” v_i and μ_t and the distributions of borrowing costs $\varepsilon_{i,t}$
- Problems: $\phi_i(r_{i,t} | \mu_t)$ is endogenous and not directly observable, β_i is heterogeneous.

Game Heterogeneity



From the point of view of the researcher, each day:

- borrowing costs are drawn from a different distribution

$$s_{i,t} = \mu_t + \varepsilon_{i,t}$$

- Then, the equilibrium quotes $r_{i,t}$ are drawn from different distributions $r_{i,t} = \rho_i(s_{i,t}; \mu_t)$

Additively Separable Quotes

- Following Haile, Hong and Shum (2003) and Krasnokutskaya (2011)
- For all bank j , let $q_{j,t} = \rho_j(\varepsilon_{j,t}; 0)$ be the equilibrium strategy when $\mu = 0$
- **Proposition:** Let $\mu \neq 0$, then for all bank j , its equilibrium strategy is

$$\rho_j(\mu + \varepsilon_j; \mu) = \mu + \rho_j(\varepsilon_j; 0)$$

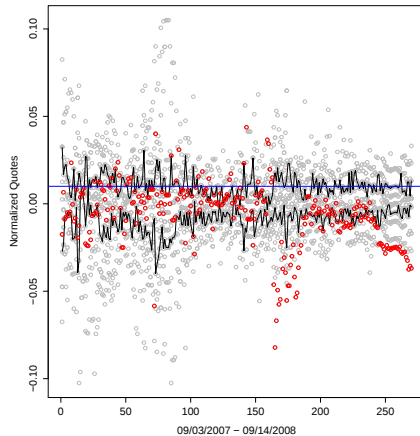
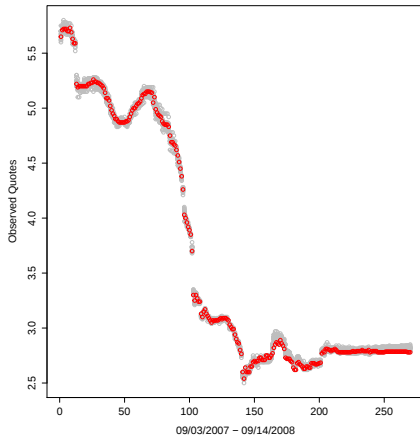
- Therefore, $r_{i,t} = \mu_t + q_{i,t}$ for all $\mu \neq 0$
- At each t , banks are playing the same BNE (although "translated" by μ_t)

- Suppose μ_t was observable
- Then $q_{i,t}$ would be identified from $q_{i,t} = r_{i,t} - \mu_t$
- The sample $\{q_t\}_{t=1}^T$ consists of independent realizations of the BNE of the game with $\mu = 0$
- Moreover,

$$\varepsilon_{i,t} = q_{i,t} - \beta_i \phi_i(q_{i,t} | \mu = 0) + v_i$$

- If the distributions of q_j are identified, $\phi_i(\cdot | \mu = 0)$ is also identified.

Normalized Game



Estimation of ϕ_i

- $\phi_i(r_{i,t}|\mu_t) = \Pr\left(r_t^{(n)} - \mu_t \leq r_{i,t} - \mu_t \leq r_t^{(\bar{n})} - \mu_t\right) = \Pr\left(q_t^{(n)} \leq q_{i,t} \leq q_t^{(\bar{n})}\right)$
- For each $j \neq i$, take T_s draws with replacement from the sample $\{\tilde{q}_{j,t}\}_{t=1}^T$
- Let $\tilde{q}_{j,\tau}^s$ denote the τ -th of such draws
- T_s possible scenarios for bank i : $\left\{\tilde{q}_{-i,\tau}^s\right\}_{\tau=1}^{T_s}$ (Hortaçsu, 2000)
- The resulting estimate is

$$\hat{\phi}_i(r_{i,t}|\mu_t) = \frac{1}{T_s} \sum_{\tau=1}^{T_s} \mathbf{1}\left(\tilde{q}_{\tau}^{s(n)} \leq \tilde{q}_{i,t} \leq \tilde{q}_{\tau}^{s(\bar{n})}\right)$$

Identification of β_i

- Let $\tilde{q}_{i,t} = q_{j,t} + k$, then

$$\varepsilon_{i,t} = \tilde{q}_{i,t} - E[\tilde{q}_i] - \beta_i(\phi_i(\tilde{q}_{i,t}) - E[\phi_i(\tilde{q}_i)])$$

- β_i can be identified by imposing additional restrictions on the distribution of $\varepsilon_{i,t}$
- If $\text{Med}(\varepsilon_{i,t}) = 0$ then

$$\beta_i = \frac{\text{Med}(\tilde{q}_i) - (E[\tilde{q}_i])}{\phi_i(\text{Med}(\tilde{q}_i)|k) - E[\phi_i(\tilde{q}_i|k)]}$$

Negative?

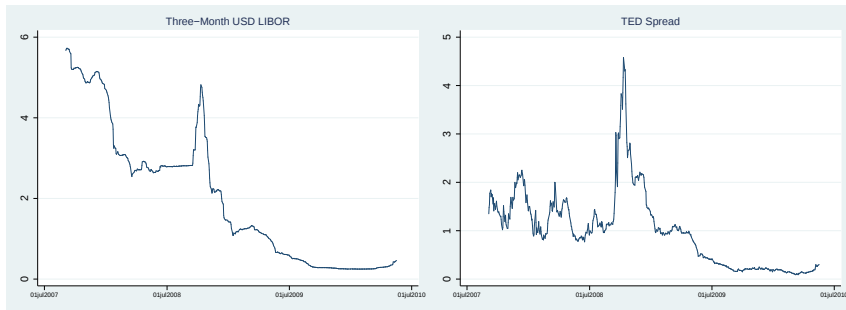
- Moreover, $v_i - \bar{v}$ is identified for all i

$$v_i - \bar{v} = E[\bar{q}] - E[\tilde{q}_i] + \beta_i E[\phi_i(\tilde{q}_i|k)] - E[\bar{\beta}\phi]$$

Unobserved Game Heterogeneity

- Possible solutions
 - Campo, Perrigne & Vuong (2003): Assume $z_t = z(\mu_t)$ where z_t is observable (e.g. the number of bidders)
 - Krasnokutskaya (2011): Recover the distribution of μ from the joint distribution of two bids
- Krasnokutskaya assumes that μ_t is independently drawn from the same distribution each period.

Non Stationary μ_t



- Augmented Dickey-Fuller test statistic: 0.553
- The 10% critical value for rejection of the null is -2.57 (for a one-sided test).
- The result is robust to several specifications of the test.

Removing the Heterogeneity

- For three banks i_1, i_2 and i_3

$$r_{i_1,t} = \mu_t + q_{i_1,t}$$

$$r_{i_2,t} = \mu_t + q_{i_2,t}$$

$$r_{i_3,t} = \mu_t + q_{i_3,t}$$

- Thus

$$r_{i_1,t} - r_{i_2,t} = -q_{i_2,t} + q_{i_1,t}$$

$$r_{i_3,t} - r_{i_2,t} = -q_{i_2,t} + q_{i_3,t}$$

- Where $r_{i_1,t} - r_{i_2,t}$ and $r_{i_3,t} - r_{i_2,t}$ are observable and $q_{i_1,t}, q_{i_2,t}$ and $q_{i_3,t}$ are mutually independent

Identification of Normalized Quotes

- Let Ψ denote the joint characteristic function of $(r_{i_1,t} - r_{i_2,t}, r_{i_3,t} - r_{i_2,t})$ and Ψ_1 its partial derivative with respect to its first argument. Then

$$\Phi_{-q_{i_2}}(s) = \exp\left(\int_0^s \frac{\Psi_1(0, u)}{\Psi(0, u)} du - isE[q_{i_1}]\right)$$

$$\Phi_{q_{i_1}}(s) = \frac{\Psi(s, 0)}{\Phi_{-q_{i_2}}(s)}$$

$$\Phi_{q_{i_3}}(s) = \frac{\Psi(0, s)}{\Phi_{-q_{i_2}}(s)}$$

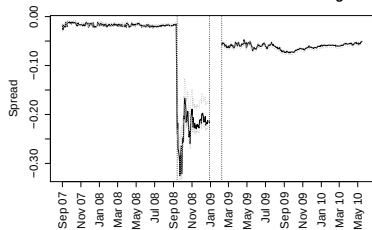
- Kotlarski (1963), Li and Vuong (1998), Li, Perrigne, and Vuong (2000), and Krasnokutskaya (2011)
- Result: The distributions of $q_{i_1} - E[q_{i_1}]$, $q_{i_2} - E[q_{i_1}]$ and $q_{i_3} - E[q_{i_1}]$ are identified
- A lower bound on μ_t is identified (although, with some noise).

Identification from Quotes Alone

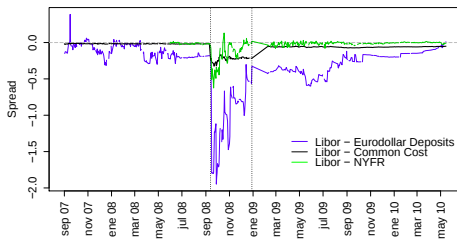
- $\beta_i = \frac{\alpha_i + v_i}{\bar{N}}$ and $v_i - \bar{v}$ are identified, for all $i \in \mathcal{N}$
- The distribution of ε_i is identified, for all $i \in \mathcal{N}$
- With noise:
 - A lower bound on μ_t
 - All realizations of $\varepsilon_{i,t}$ and $s_{i,t}$ (The true borrowing costs).

Partial identification Results

3M USD Libor – Estimated Common Borrowing Costs



3M USD Libor – Other Measures of Borrowing Costs



Other Measures of Average Borrowing Costs

- Given estimates of v_i and α_i
- The deviations from truthful reporting can be decomposed:

$$r_{i,t} - s_{i,t} = \underbrace{\frac{\alpha_i}{\tilde{N}} \phi_i(r_{i,t} | \mu_t)}_{\text{Portfolio}} + \underbrace{\frac{v_i}{\tilde{N}} \phi_i(r_{i,t} | \mu_t) - v_i}_{\text{Signaling}}$$

Determinants of Misreporting

	(i): 09/03/2007 - 09/14/2008		(ii): 09/14/2008 - 12/31/2008		(iii): 02/09/2009 - 05/17/2010	
	Signaling	Portfolio	Signaling	Portfolio	Signaling	Portfolio
BOA	-2.7*	0.2	-29.5*	1.8*	-6.9*	0.7*
BTMU	-1.3*	0.5*	-13.9*	2.4*	-0.9*	0.5*
Barclays	0.0	0.5*	0.0	3.7*	-4.0*	-2.1*
Citibank	-2.1*	-1.0*	-15.9*	-15.6*	-7.8*	0.1
Credit Suisse	-0.4	-0.7*	-13.2*	1.2	-1.7*	-2.8*
Deutsche	-2.1*	0.1	-19.0*	-0.7	-8.5*	-1.0*
HBOS	-0.6	-0.5*	-7.4*	-5.1*	-7.8*	-1.3
HSBC	-2.4*	-0.6*	-30.8*	1.8*	-10.3*	-0.1
JPM	-2.8*	0.1	-47.7*	-0.9*	-7.2*	1.2*
Lloyds	-2.3*	0.2*	-16.3*	-14.8*	0.0	-0.1
Norinchukin	-0.6*	0.3*	-19.4*	3.1*	-0.8*	-3.0*
RBC	-1.6*	-0.3	-25.2*	1.6	-0.6	-0.8*
RBS	-2.2*	-0.1	-11.5*	-1.8*	-8.5*	0.6*
Rabobank	-2.4*	-1.0*	-34.7*	-4.0*	-7.2*	0.7*
UBS	-2.4*	0.0	-26.0*	3.1*	-3.3*	-1.9*
WestLB	-1.0*	-0.0	-24.9*	2.9*	-0.7*	0.7*

Determinants of Misreporting

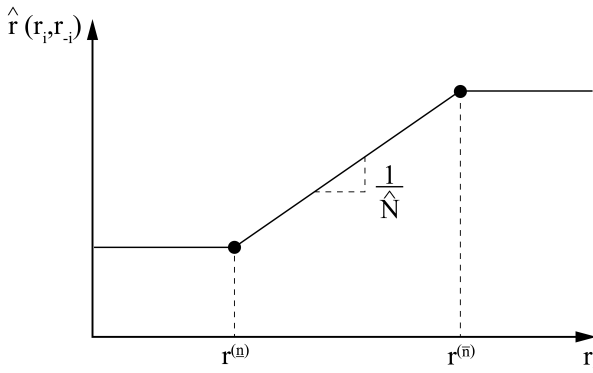
Alternative Estimation using H15 and NYFR

	(ii): 09/14/2008 - 12/31/2008				(iii): 02/09/2009 - 05/17/2010			
	Portfolio		Signaling		Portfolio		Signaling	
	H15	NYFR	H15	NYFR	H15	NYFR	H15	NYFR
BOA	-3.2*	3.1*	-90.7*	-14.1*	-1.1*	1.5*	-23.2*	-0.1
BTMU	-0.9	2.4*	-94.2*	-14.6*	0	0.6*	-25.3*	-0.1
Barclays	0.9	3.3*	-95.3*	-14.8*	-3.7*	-1.8*	-23.9*	-0.1
Citibank	-20*	-15.5*	-92.7*	-14.4*	-1.3*	0.7*	-23.8*	-0.1
Credit Suisse	-2.9*	1.1	-93.3*	-14.5*	-5.2*	-2.7*	-23.3*	-0.1
Deutsche	-7.7*	-0.2	-89.3*	-13.9*	-1.5*	-0.7*	-25.1*	-0.1
HBOS	-12.6*	-5.7*	-89.9*	-14*	-2.1	-0.9	-24.7*	-0.1
HSBC	-3.2	3.2*	-90.6*	-14.1*	-0.2*	0	-25.7*	-0.1
JPM	-1.1*	-0.8*	-97.8*	-15.2*	-1	2.2*	-22.7*	-0.1
Lloyds	-19.4*	-14.7*	-92.5*	-14.4*	-0.5*	-0.1	-25.4*	-0.1
Norinchukin	-2	3.4*	-91.7*	-14.2*	-5.4*	-2.9*	-23.3*	-0.1
RBC	-5.8*	3*	-87.7*	-13.6*	-1.9*	-0.8*	-24.7*	-0.1
RBS	-4.5*	-1.9*	-95*	-14.8*	-0.7*	1.3*	-23.8*	-0.1
Rabobank	-6*	-3.3*	-94.8*	-14.7*	-0.7	1.3*	-23.7*	-0.1
UBS	-4.4*	4.6*	-87.4*	-13.6*	-3.7*	-1.7*	-23.8*	-0.1
WestLB	-4	4.1*	-88.5*	-13.7*	-0.2*	0.7*	-24.9*	-0.1

- The paper:
 - Illustrates that methods from the empirical auctions literature can be applied to a broader class of Bayesian games
 - Establishes limits to the information that can be recovered from optimal behavior alone
 - Estimates more precise measures of borrowing costs during the financial crisis
 - Identifies banks incentives to misreport Libor, even with (nonstationary) unobserved heterogeneity
- Looking forward:
 - Might contribute to evaluate reforms to Libor regulation and to design alternative benchmark interest rates
 - Suggest that delaying publication of the quotes should increase their reliability

Marginal Effect on the Fix Rate

Back

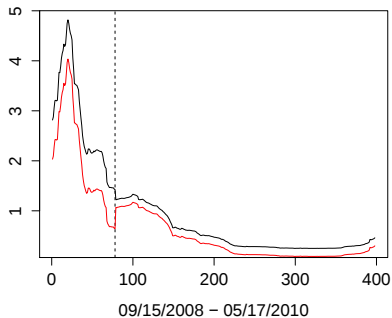


$$\phi_i(r_i|\mu) = \sum_{k=\underline{n}}^{\bar{n}-2} \sum_{\substack{|M|=k \\ i \notin M}} \left(\prod_{j \in M} F_j \left(\rho_j^{-1}(r_i) - \mu \right) \prod_{j \in M^c} \left(1 - F_j \left(\rho_j^{-1}(r_i) - \mu \right) \right) \right)$$

Nonnegative Borrowing Costs

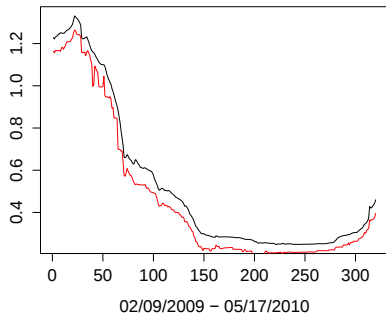
$$w_t = \text{libor}_t + \min_i \delta_i - 2 \max_i \sigma_{\varepsilon_i}$$

Libor vs Cost (Worst Case Scenario)



$$s_{i,t} = \mu_t + \delta_i + \varepsilon_{i,t}$$

Libor vs Minimum Cost (Lower Bound)



Back