

Automobile Prices in Market Equilibrium

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Utility function

- ▶ Utility derived by consumer i from consuming product j is:

$$U(\zeta_i, p_j, x_j, \xi_j; \theta)$$

Where:

ζ : vector of individual characteristics.

p : price of the product.

x : observed (by the econometrician) product attributes.

ξ : unobserved product attributes.

θ : k -vector of parameters to be estimated.

- ▶ Throughout we will take ζ to have a known distribution.
- ▶ θ include any parameters determining the distribution of consumer characteristics.

Consumer i chooses good j if and only if:

$$U(\zeta_i, p_j, x_j, \xi_j; \theta) \geq U(\zeta_i, p_r, x_r, \xi_r, \theta) \quad \text{for } r = 0, 1, \dots, J$$

The set of values for ζ that induces the choice of good "j"

$$A_j = \{ \zeta : U(\zeta, p_j, x_j, \xi_j; \theta) \geq U(\zeta, p_r, x_r, \xi_r; \theta), \text{ for } r = 0, 1, \dots, J \}$$

The market share of good "j" is:

$$s_j(p, x, \xi; \theta) = \int_{\zeta \in A_j} P_0(d\zeta)$$

If M is the number of consumers in the market, the J -vector of demands is $Ms(p, x, \xi; \theta)$.

$$U(\zeta, p_j, x_j, \xi_j; \theta) \equiv x_j\beta - \alpha p_j + \xi_j + \epsilon_{ij} \equiv \delta_j + \epsilon_{ij} \quad (1)$$

Where:

$$\delta_j = x_j\beta - \alpha p_j + \xi_j$$

ξ_j is the mean (across consumers) of the unobserved component of utility.

If ϵ 's follows an i.i.d. distribution:

$$s_j = \int_{\epsilon} \prod_{q \neq j} P(\delta_j - \delta_q + \epsilon) P(d\epsilon)$$

if the ϵ are distributed multivariate extreme value (the logit model) then there is a closed form.

- ▶ Market shares determines:
 - ▶ the cross-price elasticities between any two product.
 - ▶ the similarity in their price and demand responses to the introduction of a new third product.

Random coefficients model

Each individual have a different preference for each different observable characteristic.

$$U(\zeta_i, p_j, x_j, \xi_j; \theta) \equiv x_j \bar{\beta} - \alpha p_j + \xi_j + \sum_k \sigma_k x_{jk} \nu_{ik} + \epsilon_{ij} \equiv \delta_j + \epsilon_{ij} \quad (2)$$

Where:

$(\zeta_i, \epsilon_i) = (\nu_{i1}, \nu_{i2}, \dots, \nu_{iK}, \epsilon_{i0}, \epsilon_{i1}, \dots, \epsilon_{iJ})$ is a mean zero vector of random variables with (a known) distribution function.

$$E(\nu_{ik}^2) = 1$$

$$\delta_j = x_j \bar{\beta} - \alpha p_j + \xi_j$$

$$\mu_{ij} = \sum_k \sigma_k x_{jk} \nu_{ik} + \epsilon_{ij}$$

Random coefficients model

Given that we have additional information on the distribution of income across households:

$$u_{ij} = \alpha \log(y_i - p_j) + \mathbf{x}_j \bar{\beta} + \xi_j + \sum_k \sigma_k x_{jk} \nu_{ik} + \epsilon_{ij}$$

for $j = 1, \dots, J$.

$$u_{i0} = \alpha \log(y_i) + \xi_0 + \sigma_0 \nu_{i0} + \epsilon_{i0}$$

It allows for interactions between consumer and product characteristics.

It allows us to make use of exogenous data on the distribution of income.

Endogenous Prices

Unobserved products characteristics (style, prestige, reputation, etc) are likely to be correlated with prices.

We will assume that ξ is mean independent of some set of exogenous instruments and then derive estimators from the orthogonality conditions those assumptions imply.

COST FUNCTIONS AND THE PRICING PROBLEM OF THE MULTIPRODUCT FIRM

F firms, each of which produce some subset, \mathcal{F}_f , of the J products.

Marginal cost is independent of output levels and log linear in a vector of cost characteristics:

$$\ln (mc_j) = w_j\gamma + \omega_j$$

Where:

w : observed cost characteristics.

ω : unobserved cost characteristics.

γ : is a vector of parameters to be estimated.

w_j might be correlated with ξ_j .

THE PRICING PROBLEM OF THE MULTIPRODUCT FIRM

The profits of firm f (Π_f) are:

$$\Pi_f = \sum_{j \in \mathcal{F}_f} (p_j - mc_j) Ms_j(p, x, \xi; \theta)$$

FOC for any $j \in \mathcal{F}_f$:

$$s_j(p, x, \xi; \theta) + \sum_{r \in \mathcal{F}_f} (p_r - mc_r) \frac{\partial s_r(p, x, \xi; \theta)}{\partial p_j} = 0$$

Define a new J by J matrix, Δ , whose (j, r) element is given by:

$$\Delta_{jr} = \begin{cases} \frac{-\partial s_r}{\partial p_j}, & \text{if } r \text{ and } j \text{ are produced by the same firm} \\ 0, & \text{Otherwise} \end{cases}$$

$$s(p, x, \xi; \theta) - \Delta(p, x, \xi; \theta) [p - mc] = 0$$

THE PRICING PROBLEM OF THE MULTIPRODUCT FIRM

Solving for the price-cost markup gives

$$p = mc + \Delta(p, x, \xi; \theta)^{-1} s(p, x, \xi; \theta)$$

The markup is:

$$b(p, x, \xi; \theta) \equiv \Delta(p, x, \xi; \theta)^{-1} s(p, x, \xi; \theta)$$

Using: $\ln(mc_j) = w_j\gamma + \omega_j$, we have:

$$\ln(p - b(p, x, \xi; \theta)) = w\gamma + \omega$$

Just as in estimating demand, estimates of the parameters of (3.6) can be obtained if one assumes orthogonality conditions between ω and appropriate instruments.

Instruments

The supply and demand unobservables are mean independent of both observed product characteristics and cost shifters:

$$E[\xi_j|z] = E[\omega_j|z] = 0$$

Where $z_j = [x_j, w_j]$ $z = [z_1, z_2, \dots, z_J]$

Data set

The data set contains J vectors (x_j, w_j, p_j, q_j) and a number of households sampled, n .

$$E \left[(\xi_j, \omega_j)' (\xi_j, \omega_j) | z \right] = \Omega (z_j)$$

Estimation

Given the data on the prices and the observed characteristics of the products, any choice of a triple consisting of an observed vector of positive market shares, say s , a distribution of consumer characteristics, say P , and the parameters of the model, say θ , implies a unique sequence of estimates for the two unobserved characteristics of our product:

$$\{\xi_j(\theta, s, P), \omega_j(\theta, s, P)\}_{j=1}^J$$

- ▶ Assume, temporarily, that we can actually calculate $\{\xi_j(\theta, \mathbf{s}^0, P_0), \omega_j(\theta, \mathbf{s}^0, P_0)\}_{j=1}^J$ for alternative values of θ .

Where \mathbf{s}^0 denote the vector of shares in the underlying population.

- ▶ we cannot actually compute the disturbances generated by P_0 , but rather only from a (simulation) estimator of it.
- ▶ At $\theta = \theta_0$ our computation will reproduce the true values of the unobserved car characteristics.

$E[\xi_j|z] = E[\omega_j|z] = 0$ imply that any function of z must be uncorrelated with the vector $\{\xi(\theta, \mathbf{s}^0, P_0), \omega(\theta, \mathbf{s}^0, P_0)\}$ when that vector is evaluated at $\theta = \theta_0$.

method of moments estimator of θ_0

Let $T(z_j)$ be a 2 by 2 matrix and $H_j(z)$ be a L by 2 matrix.

$$T(z)' T(z) = \Omega(z)^{-1}$$

$$G^J(\theta) = E \left[H_j(z) T(z) \begin{pmatrix} \xi_j(\theta, s^0, P_0) \\ \omega_j(\theta, s^0, P_0) \end{pmatrix} \right]$$

$$G^J(\theta_0) = 0$$

$$G_J(\theta; s^0, P_0) = \frac{1}{J} \sum_{j=1}^J H_j(z) (z_j) \begin{pmatrix} \xi_j(\theta, s^0, P_0) \\ \omega_j(\theta, s^0, P_0) \end{pmatrix}$$

$$\min \|G_J(\theta; s^0, P_0)\|$$

method of moments estimator of θ_0

- ▶ We do not observe s^0 but just s^n .
- ▶ We will not be able to calculate $G_J(\theta; s, P_0)$, we will use the empirical distribution of ns simulations draws from P_0 :

$$\min \|G_J(\theta; s^n, P_{ns})\|$$

- ▶ Under some conditions this estimate is consistent and asymptotically normal.

Optimal instruments

$$H_j(z) = E \left[\frac{\partial \xi_j(\theta_0, \mathbf{s}^0, P_0)}{\partial \theta}, \frac{\partial \omega_j(\theta_0, \mathbf{s}^0, P_0)}{\partial \theta} \mid z \right] T(z_j)$$

$$\equiv D_j(z) T(z_j)$$

$$z_{jk}, \quad \sum_{r \neq j, r \in \mathcal{F}_f} z_{rk}, \quad \sum_{r \neq j, r \notin \mathcal{F}_f} z_{rk}$$

Computation

Computation of the moments $G_J(\theta; s^n, P_{ns})$ for different values of θ . For each θ :

Conditional (on ν) market shares:

$$f_j(\nu_i, \delta, \mathbf{p}, \mathbf{x}, \theta) = \frac{e^{\delta_j + \mu(x_j, p_j, \nu_i, \theta_2)}}{1 + \sum_{j=1}^J e^{\delta_j + \mu(x_j, p_j, \nu_i, \theta_2)}}$$

Market shares conditional only on product characteristics:

$$s_j(\mathbf{p}, \mathbf{x}, \xi, \theta, P_0) = \int f_j(\nu_i, \delta(\mathbf{x}, \mathbf{p}, \xi), \mathbf{p}, \mathbf{x}, \theta) P_0(d\nu)$$

We will use a simulation estimator of its value,

$$s_j(\mathbf{p}, \mathbf{x}, \delta, P_{ns}; \theta)$$

Computation

- ▶ We solve for δ , $s^n = s(p, x, \delta, P_{ns}; \theta)$:

$$\delta = \delta + \ln(s^n) - \ln[s(p, x, \delta, P_{ns}; \theta)]$$

- ▶ They show that:
- ▶ For any triple (s, θ, P) , such that s is in the interior of the $J + 1$ dimensional unit simplex, $\theta \in \Theta \subset R^k$, and P is a proper distribution for ν , the operator $T(s, \theta, P) : R^J \rightarrow R^J$ defined pointwise by

$$T(s, \theta, P) [\delta_j] = \delta_j + \ln(s_j) - \ln[s_j(p, x, \delta, P; \theta)]$$

is a contraction mapping.

- ▶ Given $\delta_j(s, \theta, P)$ we have:

$$\xi_j = \delta_j(s, \theta, P) - x_j \beta$$

Next we calculate the cost-side unobservable.

Simulators for Market Share

Given a set of ns pseudo-random draws from P_0 , say, (ν_1, \dots, ν_{ns}) and calculate

$$s_j(\boldsymbol{p}, \boldsymbol{x}, \boldsymbol{\xi}, \boldsymbol{\theta}, P_{ns}) \equiv \frac{1}{ns} \sum_{i=1}^{ns} f_j(\nu_i, \boldsymbol{\delta}, \boldsymbol{p}, \boldsymbol{x}, \boldsymbol{\theta})$$

The Empirical Distribution of Income

The income distribution is assumed to be lognormal and we estimate its parameters from the March Current Population Survey (CPS) for each year of our panel.

$$u_{ijt} = \alpha \log \left(e^{m_t + \hat{\sigma}_y v_{iy}} - p_{jt} \right) + x_{jt} \bar{\beta} + \xi_{jt} + \sum_k \sigma_k x_{jkt} \nu_{ik} + \epsilon_{ijt}$$

for $j = 1, \dots, J$.

$$u_{i0t} = \alpha \log \left(e^{m_t + \hat{\sigma}_y v_{iy}} \right) + \xi_{0t} + \sigma_0 \nu_{i0} + \epsilon_{i0t}$$

Minimization

This search was performed using the Nelder-Mead (1965) nonderivative "simplex" search routine.

A SAMPLE FROM 1990 OF ESTIMATED DEMAND ELASTICITIES
WITH RESPECT TO ATTRIBUTES AND PRICE
(BASED ON TABLE IV (CRTS) ESTIMATES)

Model	HP/Weight	Value of Attribute/Price Elasticity of demand with respect to:			Price
		Air	MP\$	Size	
Mazda323	0.366	0.000	3.645	1.075	5.049
	0.458	0.000	1.010	1.338	6.358
Sentra	0.391	0.000	3.645	1.092	5.661
	0.440	0.000	0.905	1.194	6.528
Escort	0.401	0.000	4.022	1.116	5.663
	0.449	0.000	1.132	1.176	6.031
Cavalier	0.385	0.000	3.142	1.179	5.797
	0.423	0.000	0.524	1.360	6.433
Accord	0.457	0.000	3.016	1.255	9.292
	0.282	0.000	0.126	0.873	4.798
Taurus	0.304	0.000	2.262	1.334	9.671
	0.180	0.000	-0.139	1.304	4.220
Century	0.387	1.000	2.890	1.312	10.138
	0.326	0.701	0.077	1.123	6.755
Maxima	0.518	1.000	2.513	1.300	13.695
	0.322	0.396	-0.136	0.932	4.845
Legend	0.510	1.000	2.388	1.292	18.944
	0.167	0.237	-0.070	0.596	4.134
TownCar	0.373	1.000	2.136	1.720	21.412
	0.089	0.211	-0.122	0.883	4.320
Seville	0.517	1.000	2.011	1.374	24.353
	0.092	0.116	-0.053	0.416	3.973
LS400	0.665	1.000	2.262	1.410	27.544
	0.073	0.037	-0.007	0.149	3.085
BMW 735i	0.542	1.000	1.885	1.403	37.490
	0.061	0.011	-0.016	0.174	3.515

A SAMPLE FROM 1990 OF ESTIMATED OWN- AND CROSS-PRICE SEMI-ELASTICITIES:
 BASED ON TABLE IV (CRTS) ESTIMATES

	Mazda 323	Nissan Sentra	Ford Escort	Chevy Cavalier	Honda Accord	Ford Taurus	Buick Century	Nissan Maxima	Acura Legend	Lincoln Town Car	Cadillac Seville	Lexus LS400	BMW 735i
323	-125.933	1.518	8.954	9.680	2.185	0.852	0.485	0.056	0.009	0.012	0.002	0.002	0.000
Sentra	0.705	-115.319	8.024	8.435	2.473	0.909	0.516	0.093	0.015	0.019	0.003	0.003	0.000
Escort	0.713	1.375	-106.497	7.570	2.298	0.708	0.445	0.082	0.015	0.015	0.003	0.003	0.000
Cavalier	0.754	1.414	7.406	-110.972	2.291	1.083	0.646	0.087	0.015	0.023	0.004	0.003	0.000
Accord	0.120	0.293	1.590	1.621	-51.637	1.532	0.463	0.310	0.095	0.169	0.034	0.030	0.005
Taurus	0.063	0.144	0.653	1.020	2.041	-43.634	0.335	0.245	0.091	0.291	0.045	0.024	0.006
Century	0.099	0.228	1.146	1.700	1.722	0.937	-66.635	0.773	0.152	0.278	0.039	0.029	0.005
Maxima	0.013	0.046	0.236	0.256	1.293	0.768	0.866	-35.378	0.271	0.579	0.116	0.115	0.020
Legend	0.004	0.014	0.083	0.084	0.736	0.532	0.318	0.506	-21.820	0.775	0.183	0.210	0.043
TownCar	0.002	0.006	0.029	0.046	0.475	0.614	0.210	0.389	0.280	-20.175	0.226	0.168	0.048
Seville	0.001	0.005	0.026	0.035	0.425	0.420	0.131	0.351	0.296	1.011	-16.313	0.263	0.068
LS400	0.001	0.003	0.018	0.019	0.302	0.185	0.079	0.280	0.274	0.606	0.212	-11.199	0.086
735i	0.000	0.002	0.009	0.012	0.203	0.176	0.050	0.190	0.223	0.685	0.215	0.336	-9.376