Princeton-Santa Barbara Workshop on Modern Power Grids:

Stochastic, Statistical and Optimization Models

◇

Mixing Dynamic Programming and Spatial Decomposition Methods

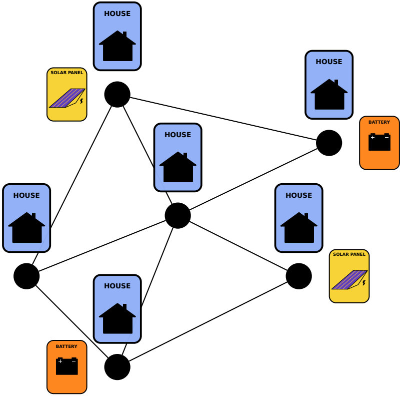
P. Carpentier — J.-P. Chancelier — M. De Lara — F. Pacaud

ENSTA Paris — ENPC ParisTech — Efficacity

Princeton-Santa Barbara Workshop, June 14-15, 2021

Motivation

We consider a *peer-to-peer* microgrid where houses exchange energy, and we formulate it as a large-scale stochastic optimization problem



How to manage such network in an (almost) optimal way?

Motivation

We will show that, for a large district microgid with

- 48 buildings
- 16 batteries
- 71 edges network

methods mixing temporal decomposition (dynamic programming) and spatial decomposition (price or resource allocation) give better results than the standard SDDP algorithm

- in terms of CPU time: $\times 3$ faster

SDDP CPU time: 453'	Decomp CPU time: 128'

- in terms of cost gap: 1.5% better

SDDP policy cost: 3550	Decomp policy cost: 3490

Lecture outline

Tools for mixing spatial and temporal decomposition methods
- Upper and lower bounds using spatial decomposition
- Temporal decomposition using dynamic programming
- The case of deterministic coordination processes

Application to the management of urban microgrids
- Nodal decomposition of a network optimization problem
- Numerical results on urban microgrids of increasing size

Another point of view: decentralized information structure
- Centralized versus decentralized information structures
- Bounds for the centralized and decentralized information structures
- Information gap

Conclusion

Tools for mixing spatial and temporal decomposition methods

- Upper and lower bounds using spatial decomposition
- Temporal decomposition using dynamic programming
- The case of deterministic coordination processes

Application to the management of urban microgrids

- Nodal decomposition of a network optimization problem
- Numerical results on urban microgrids of increasing size

Another point of view: decentralized information structure

- Centralized versus decentralized information structures
- Bounds for the centralized and decentralized information structures
- Information gap

Conclusion

An abstract optimization problem

We consider the following optimization problem

$$V_0^\sharp = \min_{u^1 \in \mathcal{U}^1_{\mathrm{ad}}, \cdots, u^N \in \mathcal{U}^N_{\mathrm{ad}}} \sum_{i=1}^{N} J^i(u^i)$$

$$\text{s.t.} \quad \underbrace{\big(\Theta^1(u^1), \cdots, \Theta^N(u^N)\big) \in \mathcal{R}_{\mathrm{ad}}}_{\text{coupling constraint}}$$

- $u^i \in \mathcal{U}^i$ a local decision variable
- $J^i : \mathcal{U}^i \to \mathbb{R}, \; i \in [\![1, N]\!]$ a local objective
- $\mathcal{U}^i_{\mathrm{ad}}$ a subset of $\mathcal{U}^i$ representing local constraints
- $\Theta^i : \mathcal{U}^i \to \mathcal{R}^i$ maps local decisions into local resources
- $\mathcal{R}_{\mathrm{ad}} \subset \mathcal{R}^1 \times \cdots \times \mathcal{R}^N$ a subset representing coupling resources constraints between units

Prices and resources are paired

- Each resource space $\mathcal{R}^i$ is in bilinear pairing with a price space $\mathcal{P}^i$
- The product spaces $\mathcal{R} = \mathcal{R}^1 \times \cdots \times \mathcal{R}^N$ and $\mathcal{P} = \mathcal{P}^1 \times \cdots \times \mathcal{P}^N$ are then paired with

$$\langle p \, , r \rangle = \sum_{i=1}^{N} \langle p^i \, , r^i \rangle$$

- We denote by $\mathcal{R}_{\mathrm{ad}}^{o}$ the polar cone of $\mathcal{R}_{\mathrm{ad}}$

$$\mathcal{R}_{\mathrm{ad}}^{o} = \big\{ p \in \mathcal{P} \mid \langle p \, , r \rangle \leq 0 \, , \ \ \forall r \in \mathcal{R}_{\mathrm{ad}} \big\}$$

Price and resource value functions

For each $i \in [\![1, N]\!]$

- for any price $p^i \in \mathcal{P}^i$, we define the local price value

$$\underline{V}_0^i[p^i] = \min_{u^i \in \mathcal{U}_{\mathrm{ad}}^i} J^i(u^i) + \langle p^i \,, \Theta^i(u^i) \rangle$$

- for any resource $r^i \in \mathcal{R}^i$, we define the local resource value

$$\overline{V}_0^i[r^i] = \min_{u^i \in \mathcal{U}_{\mathrm{ad}}^i} J^i(u^i) \quad \text{s.t.} \quad \Theta^i(u^i) = r^i$$

Theorem 1 (Upper and lower bounds for optimal value)

- *For any admissible price* $p = (p^1, \cdots, p^N) \in \mathcal{R}_{ad}^o$
- *For any admissible resource* $r = (r^1, \cdots, r^N) \in \mathcal{R}_{ad}$

we have that

$$\sum_{i=1}^N \underline{V}_0^i[p^i] \;\leq\; V_0^\sharp \;\leq\; \sum_{i=1}^N \overline{V}_0^i[r^i]$$

Tools for mixing spatial and temporal decomposition methods

Upper and lower bounds using spatial decomposition

Temporal decomposition using dynamic programming

The case of deterministic coordination processes

Application to the management of urban microgrids

Nodal decomposition of a network optimization problem

Numerical results on urban microgrids of increasing size

Another point of view: decentralized information structure

Centralized versus decentralized information structures

Bounds for the centralized and decentralized information structures

Information gap

Conclusion

The case of multistage stochastic optimization

$$V_0^\sharp(x_0^1, \dots, x_0^N) = \min_{\boldsymbol{X},\boldsymbol{U}} \mathbb{E}\Bigg(\sum_{i=1}^{N} \sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + K^i(\boldsymbol{X}_T^i) \Bigg)$$

$$\text{s.t. } \boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) \ , \quad \boldsymbol{X}_0^i = x_0^i$$

$$\forall i \in [\![1, N]\!]$$

$$\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_t)$$

$$\forall i \in [\![1, N]\!] \ , \ \forall t \in [\![0, T-1]\!]$$

$$\big(\Theta_t^1(\boldsymbol{X}_t^1, \boldsymbol{U}_t^1), \dots, \Theta_t^N(\boldsymbol{X}_t^N, \boldsymbol{U}_t^N)\big) \in \mathcal{R}_{\mathrm{ad}}$$

The case of multistage stochastic optimization

In this case, the abstract local price value

$$\underline{V}_0^i[p^i] = \min_{u^i \in \mathcal{U}^i_{\mathrm{ad}}} J^i(u^i) + \left\langle p^i \, , \Theta^i(u^i) \right\rangle$$

corresponds to a stochastic optimal control problem

$$\underline{V}_0^i[\boldsymbol{P}^i](x_0^i) = \min_{\boldsymbol{X}^i, \boldsymbol{U}^i} \mathbb{E}\left(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + \left\langle \boldsymbol{P}_t^i \, , \Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i) \right\rangle + K^i(\boldsymbol{X}_T^i) \right)$$

$$\text{s.t. } \boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) \, , \quad \boldsymbol{X}_0^i = x_0^i$$

$$\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_t)$$

This local control problem can be solved by Dynamic Programming (DP)

- if the noise process $\boldsymbol{W}$ is a white noise process
- and the price process $\boldsymbol{P}$ follows a dynamics

DP leads to a collection $\left\{ \underline{V}_t^i[\boldsymbol{P}^i] \right\}_{t \in [\![0,T]\!]}$ of local price value functions

The case of multistage stochastic optimization

In this case, the abstract local price value

$$\underline{V}_0^i[p^i] = \min_{u^i \in \mathcal{U}^i_{\mathrm{ad}}} J^i(u^i) + \left\langle p^i \,, \Theta^i(u^i) \right\rangle$$

corresponds to a stochastic optimal control problem

$$\underline{V}_0^i[\boldsymbol{P}^i](x_0^i) = \min_{\boldsymbol{X}^i, \boldsymbol{U}^i} \mathbb{E}\bigg(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + \left\langle \boldsymbol{P}_t^i \,, \Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i) \right\rangle + K^i(\boldsymbol{X}_T^i) \bigg)$$

$$\text{s.t. } \boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) \,, \quad \boldsymbol{X}_0^i = x_0^i$$

$$\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_t)$$

This local control problem can be solved by Dynamic Programming (DP)

- if the noise process $\boldsymbol{W}$ is a white noise process
- and the price process $\boldsymbol{P}$ follows a dynamics

DP leads to a collection $\left\{ \underline{V}_t^i[\boldsymbol{P}^i] \right\}_{t \in [\![0, T]\!]}$ of local price value functions

The case of multistage stochastic optimization

In the same way, the abstract local resource value

$$\overline{V}_0^i[r^i] = \min_{u^i \in \mathcal{U}_{\mathrm{ad}}^i} J^i(u^i) \quad \text{s.t.} \quad \Theta^i(u^i) = r^i$$

corresponds to a stochastic optimal control problem

$$\begin{aligned}
\overline{V}_0^i[\boldsymbol{R}^i](x_0^i) = \min_{\boldsymbol{X}^i, \boldsymbol{U}^i} \mathbb{E}\Big(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + K^i(\boldsymbol{X}_T^i) \Big) \\
\text{s.t. } \boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) \ , \ \ \boldsymbol{X}_0^i = x_0^i \\
\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_t) \\
\Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i) = \boldsymbol{R}_t^i
\end{aligned}$$

Mix of spatial and temporal decompositions

For any admissible coordination price process $\boldsymbol{P} \in \mathcal{R}^{o}_{\mathrm{ad}}$
and for any admissible coordination resource process $\boldsymbol{R} \in \mathcal{R}_{\mathrm{ad}}$,
we have bounds of the optimal value $V_0^\sharp$

$$\sum_{i=1}^{N} \underline{V}_0^i[\boldsymbol{P}^i](x_0^i) \;\leq\; V_0^\sharp \;\leq\; \sum_{i=1}^{N} \overline{V}_0^i[\boldsymbol{R}^i](x_0^i)$$

1. To obtain the bounds, we perform spatial decompositions giving
   - a collection $\left\{\underline{V}_0^i[P^i](x_0^i)\right\}_{i\in[\![1,N]\!]}$ of price local values
   - a collection $\left\{\overline{V}_0^i[R^i](x_0^i)\right\}_{i\in[\![1,N]\!]}$ of resource local values

   The computation of these local values can be performed in parallel

2. To compute each local value, we perform temporal decomposition based on Dynamic Programming: for each local unit i, we obtain
   - a sequence $\left\{\underline{V}_t^i[P^i]\right\}_{t\in[\![0,T]\!]}$ of price local value functions
   - a sequence $\left\{\overline{V}_t^i[R^i]\right\}_{t\in[\![0,T]\!]}$ of resource local value functions

   The computation of these local values functions is done sequentially

Mix of spatial and temporal decompositions

For any admissible coordination price process $\boldsymbol{P} \in \mathcal{R}^{o}_{\mathrm{ad}}$
and for any admissible coordination resource process $\boldsymbol{R} \in \mathcal{R}_{\mathrm{ad}}$,
we have bounds of the optimal value $V_0^\sharp$

$$\sum_{i=1}^{N} \underline{V}_0^i[\boldsymbol{P}^i](x_0^i) \;\leq\; V_0^\sharp \;\leq\; \sum_{i=1}^{N} \overline{V}_0^i[\boldsymbol{R}^i](x_0^i)$$

1. To obtain the bounds, we perform spatial decompositions giving
   - a collection $\left\{\underline{V}_0^i[\boldsymbol{P}^i](x_0^i)\right\}_{i \in [\![1,N]\!]}$ of price local values
   - a collection $\left\{\overline{V}_0^i[\boldsymbol{R}^i](x_0^i)\right\}_{i \in [\![1,N]\!]}$ of resource local values

   The computation of these local values can be performed in parallel

2. To compute each local value, we perform temporal decomposition based on Dynamic Programming: for each local unit i, we obtain
   - a sequence $\left\{\underline{V}_t^i[\boldsymbol{P}^i]\right\}_{t \in [\![0,T]\!]}$ of price local value functions
   - a sequence $\left\{\overline{V}_t^i[\boldsymbol{R}^i]\right\}_{t \in [\![0,T]\!]}$ of resource local value functions

   The computation of these local values functions is done sequentially

Mix of spatial and temporal decompositions

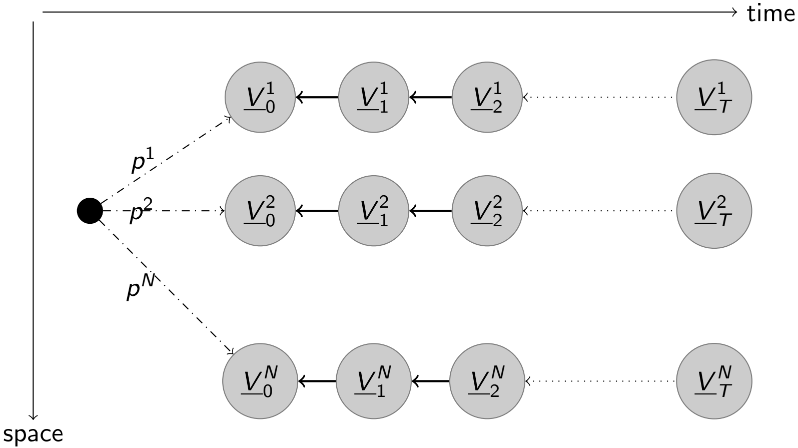


Figure: The case of price decomposition

The case of deterministic price and resource processes

We assume that $\boldsymbol{W}$ is a white noise process, and we restrict ourselves to **deterministic** admissible coordination processes $p \in \mathcal{R}^o_{\mathrm{ad}}$ and $r \in \mathcal{R}_{\mathrm{ad}}$

- The local value functions $\underline{V}^i_t[p^i]$ and $\overline{V}^i_t[r^i]$ are easy to compute because they only depend on the local state variable x^i
- It is easy to obtain tighter bounds by maximizing the lower bound w.r.t. prices and minimizing the upper bound w.r.t. resources

$$\sup_{p \in \mathcal{R}^o_{\mathrm{ad}}} \sum_{i=1}^{N} \underline{V}^i_0[p^i](x^i_0) \ \leq \ V^\sharp_0 \ \leq \ \inf_{r \in \mathcal{R}_{\mathrm{ad}}} \sum_{i=1}^{N} \overline{V}^i_0[r^i](x^i_0)$$

The case of deterministic price and resource processes

We assume that $\boldsymbol{W}$ is a white noise process, and we restrict ourselves to **deterministic** admissible coordination processes $p \in \mathcal{R}^o_{\mathrm{ad}}$ and $r \in \mathcal{R}_{\mathrm{ad}}$

The local value functions $\underline{V}^i_t[p^i]$ and $\overline{V}^i_t[r^i]$ allow the computation of global policies by solving (online) static optimization problems

- In the case of local price value functions, the policy is obtained as

$$\underline{\gamma}_t(x^1_t, \cdots, x^N_t) \in \underset{u^1_t, \cdots, u^N_t}{\arg\min} \ \mathbb{E}\bigg(\sum_{i=1}^N L^i_t(x^i_t, u^i_t, \boldsymbol{W}_{t+1}) + \sum_{i=1}^N \underline{V}^i_{t+1}[p^i](\boldsymbol{X}^i_{t+1}) \bigg)$$
$$\text{s.t.} \ \ \boldsymbol{X}^i_{t+1} = g^i_t(x^i_t, u^i_t, \boldsymbol{W}_{t+1}) \ , \ \ \forall i \in [\![1, N]\!]$$
$$\big(\Theta_t(x^1_t, u^1_t), \cdots, \Theta_t(x^N_t, u^N_t)\big) \in \mathcal{R}_{\mathrm{ad}}$$

- A global policy based on resource value functions is also available

Estimating the expected cost of such policies by Monte Carlo simulation leads to a statistical upper bound of the optimal cost of the problem

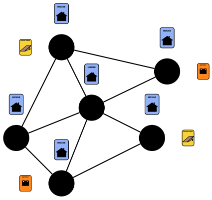
Progress status

- First, we have obtained lower and upper bounds
  for a global optimization problem with coupling constraints
  thanks to two spatial decomposition schemes
  - price decomposition
  - resource decomposition

- Second, we have computed the lower and upper bounds
  by dynamic programming (temporal decomposition)

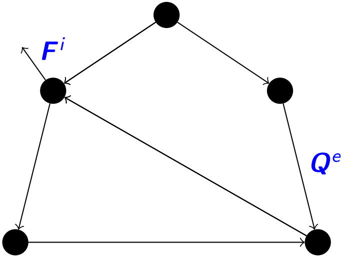
- Using the price and resource Bellman value functions,
  we have devised two online policies for the global problem

- Now, we apply these decomposition schemes
  to large-scale network problems



Network and flows

Directed graph $G = (\mathcal{V}, \mathcal{E})$



- $\boldsymbol{Q}_t^e$ flow through edge e,
- $\boldsymbol{F}_t^i$ flow imported at node i

Let A be the *node-edge* incidence matrix

Each node corresponds to a building with its own devices (battery, hot water tank, solar panel. . .)

At each time $t \in [\![0, T-1]\!]$, the Kirchhoff current law couples node and edge flows

$$A\boldsymbol{Q}_t + \boldsymbol{F}_t = 0$$

Optimization problem at a given node

At each **node** $i \in \mathcal{V}$, given a node flow process $\boldsymbol{F}^i$, we minimize the house cost

$$J^i_{\mathcal{V}}(\boldsymbol{F}^i) = \min_{\boldsymbol{X}^i, \boldsymbol{U}^i} \mathbb{E}\left(\sum_{t=0}^{T-1} L^i_t(\boldsymbol{X}^i_t, \boldsymbol{U}^i_t, \boldsymbol{W}^i_{t+1}) + K^i(\boldsymbol{X}^i_T) \right)$$

subject to, for all $t \in [\![0, T-1]\!]$

i) nodal dynamics constraints (battery, hot water tank)

$$\boldsymbol{X}^i_{t+1} = g^i_t(\boldsymbol{X}^i_t, \boldsymbol{U}^i_t, \boldsymbol{W}^i_{t+1})$$

ii) nonanticipativity constraints (future remains unknown)

$$\sigma(\boldsymbol{U}^i_t) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_{t+1})$$

iii) nodal load balance equations (demand - production = import)

$$\Delta^i_t(\boldsymbol{X}^i_t, \boldsymbol{U}^i_t, \boldsymbol{W}^i_{t+1}) = \boldsymbol{F}^i_t$$

Remarks

- Local noise $\boldsymbol{W}^i_t$ in the formulation of problem at node i
- Global noise $\boldsymbol{W}_{t+1} = (\boldsymbol{W}^1_{t+1}, \ldots, \boldsymbol{W}^N_{t+1})$ in the nonanticipativity constraint

Transportation cost and global optimization problem

We define the network cost as the sum over time and **edges** of the costs of flows $\boldsymbol{Q}_t^e$ through the edges of the network

$$J_{\mathcal{E}}(\boldsymbol{Q}) = \mathbb{E}\left(\sum_{t=0}^{T-1} \sum_{e \in \mathcal{E}} l_t^e(\boldsymbol{Q}_t^e) \right)$$

This transportation cost is additive in space, in time and in uncertainty!

The global optimization problem is obtained by gathering all elements

$$V_0^\sharp \ = \ \min_{\boldsymbol{F}, \boldsymbol{Q}} \ \sum_{i \in \mathcal{V}} J_{\mathcal{V}}^i(\boldsymbol{F}^i) + J_{\mathcal{E}}(\boldsymbol{Q})$$

$$\text{s.t. } A\boldsymbol{Q} + \boldsymbol{F} = 0$$

Price and resource decompositions

- Price problem

$$\underline{V}_0[P] = \min_{F,Q} \sum_{i \in \mathcal{V}} J^i_{\mathcal{V}}(F^i) + J_{\mathcal{E}}(Q) + \langle P , AQ + F \rangle$$

$$= \sum_{i \in \mathcal{V}} \underbrace{\Big(\min_{F_i} J^i_{\mathcal{V}}(F^i) + \langle P^i , F^i \rangle \Big)}_{\text{Node } i\text{'s subproblem}} + \underbrace{\Big(\min_{Q} J_{\mathcal{E}}(Q) + \langle A^\top P , Q \rangle \Big)}_{\text{Network subproblem}}$$

- Resource problem

$$\overline{V}_0[R] = \min_{F,Q} \sum_{i \in \mathcal{V}} J^i_{\mathcal{V}}(F^i) + J_{\mathcal{E}}(Q) \quad \text{s.t.} \quad AR + F = 0 , \quad Q = R$$

$$= \sum_{i \in \mathcal{V}} \Big(\min_{F_i} J^i_{\mathcal{V}}(F^i) \ \text{s.t.} \ F^i = -(AR)^i \Big) + \Big(\min_{Q} J_{\mathcal{E}}(Q) \ \text{s.t.} \ Q = R \Big)$$

Objective

Find **deterministic** processes $\widehat{p}$ and $\widehat{r}$ with a gap as small as possible

$$\sup_{p} \underline{V}_0[p] \leq V_0^\sharp \leq \inf_{r} \overline{V}_0[r]$$

Price and resource decompositions

- Price problem

$$\underline{V}_0[\boldsymbol{P}] = \min_{\boldsymbol{F},\boldsymbol{Q}} \sum_{i\in\mathcal{V}} J^i_{\mathcal{V}}(\boldsymbol{F}^i) + J_{\mathcal{E}}(\boldsymbol{Q}) + \langle \boldsymbol{P} , A\boldsymbol{Q} + \boldsymbol{F} \rangle$$

$$= \sum_{i\in\mathcal{V}} \underbrace{\Big(\min_{\boldsymbol{F}_i} J^i_{\mathcal{V}}(\boldsymbol{F}^i) + \langle \boldsymbol{P}^i , \boldsymbol{F}^i \rangle \Big)}_{\text{Node } i\text{'s subproblem}} + \underbrace{\Big(\min_{\boldsymbol{Q}} J_{\mathcal{E}}(\boldsymbol{Q}) + \langle A^\top \boldsymbol{P} , \boldsymbol{Q} \rangle \Big)}_{\text{Network subproblem}}$$

- Resource problem

$$\overline{V}_0[\boldsymbol{R}] = \min_{\boldsymbol{F},\boldsymbol{Q}} \sum_{i\in\mathcal{V}} J^i_{\mathcal{V}}(\boldsymbol{F}^i) + J_{\mathcal{E}}(\boldsymbol{Q}) \quad \text{s.t.} \quad A\boldsymbol{R} + \boldsymbol{F} = 0 \,, \quad \boldsymbol{Q} = \boldsymbol{R}$$

$$= \sum_{i\in\mathcal{V}} \Big(\min_{\boldsymbol{F}_i} J^i_{\mathcal{V}}(\boldsymbol{F}^i) \ \text{s.t.} \ \boldsymbol{F}^i = -(A\boldsymbol{R})^i \Big) + \Big(\min_{\boldsymbol{Q}} J_{\mathcal{E}}(\boldsymbol{Q}) \ \text{s.t.} \ \boldsymbol{Q} = \boldsymbol{R} \Big)$$

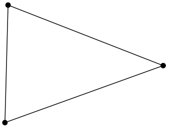
Objective

Find **deterministic** processes $\widehat{p}$ and $\widehat{r}$ with a gap as small as possible

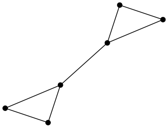
$$\sup_p \underline{V}_0[p] \leq V_0^\sharp \leq \inf_r \overline{V}_0[r]$$

Different urban configurations

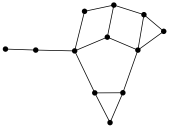
3-Nodes

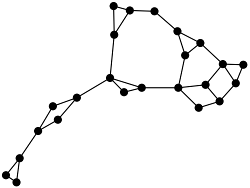


6-Nodes

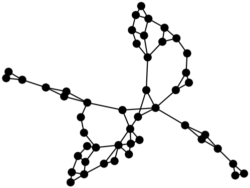


12-Nodes





24-Nodes



48-Nodes

Problem settings

Thanks to the periodicity of demands and electricity tariffs,
the microgrid management problem can be solved day by day

- One day horizon with a 15mn time step: $T = 96$
- Weather corresponds to a sunny day in Paris *(June 28, 2015)*
- We mix three kinds of buildings
  1. battery + electrical hot water tank
  2. solar panel + electrical hot water tank
  3. electrical hot water tank

  and suppose that all consumers are commoners sharing their devices

Algorithms implemented on the problem

SDDP

We use the SDDP algorithm to solve the problem globally

- but noises $\boldsymbol{W}_t^1, \cdots, \boldsymbol{W}_t^N$ are independent node by node, so that the support size of the noise may be huge ($|\mathrm{supp}(\boldsymbol{W}_t^i)|^N$)
- and thus we must resample the noise to be able to compute the cuts

Price decomposition

Spatial decomposition and maximization w.r.t. a deterministic price p

- Each nodal subproblem solved by a DP-like method
- Maximisation w.r.t. p by Quasi-Newton (BFGS) method

$$p^{(k+1)} = p^{(k)} + \rho^{(k)} H^{(k)} \nabla \underline{V}_0[p^{(k)}]$$

- Oracle $\nabla \underline{V}_0[p]$ estimated by Monte Carlo ($N^{scen} = 1,000$)

Resource decomposition

Spatial decomposition and minimization w.r.t. a deterministic resource process r

Exact upper and lower bounds on the global problem

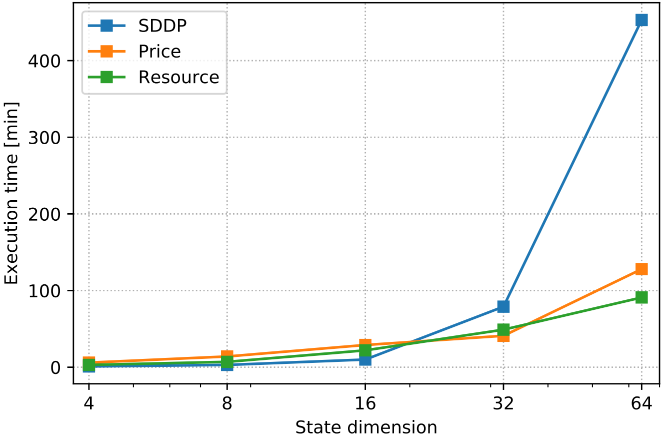
	Network	3-Nodes	6-Nodes	12-Nodes	24-Nodes	48-Nodes
State dim.	$\|\mathbb{X}\|$	4	8	16	32	64
SDDP	time	1'	3'	10'	79'	453'
SDDP	LB	225.2	455.9	889.7	1752.8	3310.3
Price	time	6'	14'	29'	41'	128'
Price	LB	213.7	447.3	896.7	1787.0	3396.4
Resource	time	3'	7'	22'	49'	91'
Resource	UB	253.9	527.3	1053.7	2105.4	4016.6

For the **48-Nodes** microgrid

- **price decomposition is more than 3 times faster than SDDP**
- price decomposition gives a (slightly) better exact lower bound than SDDP

$$\underbrace{3310.3}_{\underline{V}_0[sddp]} \leq \underbrace{3396.4}_{\underline{V}_0[price]} \leq V_0^\sharp \leq \underbrace{4016.6}_{\overline{V}_0[resource]}$$

Increase in execution time with state dimension



Policy evaluation by Monte Carlo (1,000 scenarios)

	3-Nodes	6-Nodes	12-Nodes	24-Nodes	48-Nodes
SDDP policy	226 ± 0.6	471 ± 0.8	936 ± 1.1	1859 ± 1.6	3550 ± 2.3
Price policy	228 ± 0.6	464 ± 0.8	923 ± 1.2	1839 ± 1.6	3490 ± 2.3
Gap	+0.9 %	-1.5%	-1.4%	-1.1%	-1.7%
Resource policy	229 ± 0.6	471 ± 0.8	931 ± 1.1	1856 ± 1.6	3503 ± 2.2
Gap	+1.3 %	0.0%	-0.5%	-0.2%	-1.2%

All the cost values above are statistical upper bounds of $V_0^\sharp$

For the **48-Nodes** microgrid

- price policy beats SDDP policy and resource policy

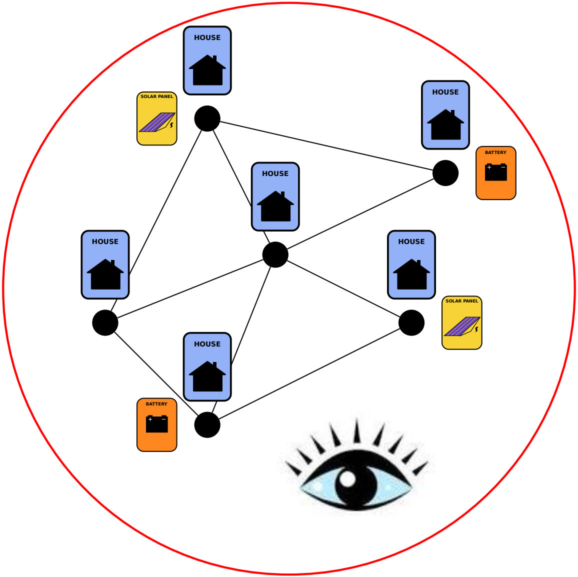
$$V_0^\sharp \leq \underbrace{3490}_{C[price]} \leq \underbrace{3503}_{C[resource]} \leq \underbrace{3550}_{C[sddp]}$$

- the exact upper bound given by resource decomposition is not so tight

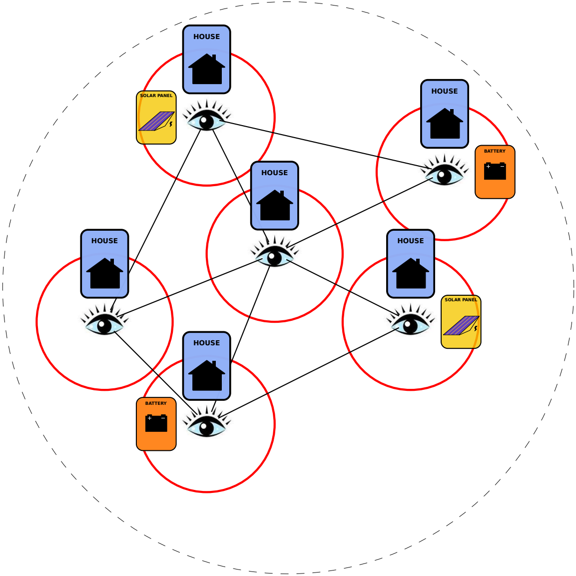
$$\underbrace{3396.4}_{\underline{V}_0[price]} \leq V_0^\sharp \leq \underbrace{3490}_{C[price]} \leq \underbrace{3503}_{C[resource]} \leq \underbrace{4016.6}_{\overline{V}_0[resource]}$$

gap <3% ≈ 3% >18%

Motivation for decentralized information



Motivation for decentralized information



Centralized information structure

Up to now, we have studied the following centralized problem

$$V_0^{\mathrm{C}} = \min_{\boldsymbol{F},\boldsymbol{Q}} \Bigg(\sum_{i\in\mathcal{V}} \underbrace{\min_{\boldsymbol{X}^i,\boldsymbol{U}^i} \mathbb{E}\Big(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) + K^i(\boldsymbol{X}_T^i) \Big)}_{J_{\mathcal{V}}^i(\boldsymbol{F}^i)} + \underbrace{\mathbb{E}\Big(\sum_{t=0}^{T-1} \sum_{e\in\mathcal{E}} l_t^e(\boldsymbol{Q}_t^e) \Big)}_{J_{\mathcal{E}}(\boldsymbol{Q})} \Bigg)$$

subject to, for all $t \in [\![0, T-1]\!]$ and for all $i \in \mathcal{V}$

$$A\boldsymbol{Q}_t + \boldsymbol{F}_t = 0 \qquad \text{(network constraints)}$$

$$\boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) \qquad \text{(nodal dynamic constraints)}$$

$$\Delta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) = \boldsymbol{F}_t^i \qquad \text{(nodal balance equation)}$$

$$\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_{t+1}) \qquad \text{(information constraints)}$$

with $\boldsymbol{W}_t = (\boldsymbol{W}_t^1, \ldots, \boldsymbol{W}_t^N)$ global noise process

Decentralized information structure

Consider now the following decentralized problem

$$V_0^{\mathrm{D}} = \min_{\boldsymbol{F},\boldsymbol{Q}} \Bigg(\sum_{i\in\mathcal{V}} \underbrace{\min_{\boldsymbol{X}^i,\boldsymbol{U}^i} \mathbb{E}\bigg(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) + K^i(\boldsymbol{X}_T^i) \bigg)}_{J_{\mathcal{V}}^i(\boldsymbol{F}^i)} + \underbrace{\mathbb{E}\bigg(\sum_{t=0}^{T-1} \sum_{e\in\mathcal{E}} l_t^e(\boldsymbol{Q}_t^e) \bigg)}_{J_{\mathcal{E}}(\boldsymbol{Q})} \Bigg)$$

subject to, for all $t \in [\![0, T-1]\!]$ and for all $i \in \mathcal{V}$

$$A\boldsymbol{Q}_t + \boldsymbol{F}_t = 0 \qquad \text{(network constraints)}$$

$$\boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) \qquad \text{(nodal dynamic constraints)}$$

$$\Delta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) = \boldsymbol{F}_t^i \qquad \text{(nodal balance equation)}$$

$$\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_0^i, \cdots, \boldsymbol{W}_{t+1}^i) \qquad \text{(information constraints)}$$

that is, the local control $\boldsymbol{U}_t^i$ is a function of local noise $(\boldsymbol{W}_0^i, \ldots, \boldsymbol{W}_{t+1}^i)$

Upper and lower decomposed bounds are not sensitive to the centralized or decentralized structures

$$\underline{V}_0[p] = \sum_{i \in \mathcal{V}} V_0^i[p^i] + V_0^{\mathcal{E}}[p] \qquad \text{with}$$

$$V_0^i[p^i] = \min_{\boldsymbol{X}^i, \boldsymbol{U}^i, \boldsymbol{F}^i} \mathbb{E}\left[\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) + \langle p_t^i , \boldsymbol{F}_t^i \rangle + K^i(\boldsymbol{X}_T^i) \right]$$

$$\text{s.t. } \boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) \ , \ \ \boldsymbol{X}_0^i = x_0^i$$

$$\Delta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) = \boldsymbol{F}_t^i$$

$$\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_1, \ldots, \boldsymbol{W}_{t+1})$$

Upper and lower decomposed bounds are not sensitive to the centralized or decentralized structures

$$\underline{V}_0[p] = \sum_{i \in \mathcal{V}} V_0^i[p^i] + V_0^{\mathcal{E}}[p] \qquad \text{with}$$

$$V_0^i[p^i] = \min_{\boldsymbol{X}^i, \boldsymbol{U}^i, \boldsymbol{F}^i} \mathbb{E}\Big[\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) + \langle p_t^i \,, \boldsymbol{F}_t^i \rangle + K^i(\boldsymbol{X}_T^i) \Big]$$

$$\text{s.t. } \boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) \,, \ \ \boldsymbol{X}_0^i = x_0^i$$

$$\Delta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) = \boldsymbol{F}_t^i$$

$$\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_1^i, \ldots, \boldsymbol{W}_{t+1}^i)$$

Replacing the global σ-field $\sigma(\boldsymbol{W}_1, \ldots, \boldsymbol{W}_{t+1})$ by the local σ-field $\sigma(\boldsymbol{W}_1^i, \ldots, \boldsymbol{W}_{t+1}^i)$ does not make any change in this local subproblem

The lower bound $\underline{V}_0[p]$ is the same for both information structures

A similar conclusion holds true for the upper bound $\overline{V}_0[r]$

Bounds for the centralized/decentralized cases

- Since $\boldsymbol{W}_t = (\boldsymbol{W}_t^1, \dots, \boldsymbol{W}_t^N)$, we have the inclusion of σ-fields

$$\sigma(\boldsymbol{W}_0^i, \dots, \boldsymbol{W}_t^i) \subset \sigma(\boldsymbol{W}_0, \dots, \boldsymbol{W}_t) , \ \forall i$$

We deduce that the admissible control set in case of a decentralized information structure is smaller that the one in case of a centralized information structure, so that we get

$$\underbrace{V_0^{\mathrm{C}}}_{\text{centralized}} \quad \leq \quad \underbrace{V_0^{\mathrm{D}}}_{\text{decentralized}}$$

- Finally, we obtain the following sequence of inequalities

$$\sup_{p} \underline{V}_0[p] \ \leq \ V_0^{\mathrm{C}} \ \leq \ V_0^{\mathrm{D}} \ \leq \ \inf_{r} \overline{V}_0[r]$$

Bounds for the centralized/decentralized cases

- Since $\boldsymbol{W}_t = (\boldsymbol{W}_t^1, \dots, \boldsymbol{W}_t^N)$, we have the inclusion of σ-fields

$$\sigma(\boldsymbol{W}_0^i, \dots, \boldsymbol{W}_t^i) \subset \sigma(\boldsymbol{W}_0, \dots, \boldsymbol{W}_t) , \ \forall i$$

  We deduce that the admissible control set in case of a decentralized information structure is smaller that the one in case of a centralized information structure, so that we get

$$\underbrace{V_0^{\mathrm{C}}}_{\text{centralized}} \leq \underbrace{V_0^{\mathrm{D}}}_{\text{decentralized}}$$

- Finally, we obtain the following sequence of inequalities

$$\sup_p \underline{V}_0[p] \leq V_0^{\mathrm{C}} \leq V_0^{\mathrm{D}} \leq \inf_r \overline{V}_0[r]$$

Bounds for the centralized/decentralized cases

- We have obtained

$$\overbrace{\sup_p \underline{V}_0[p] \leq V_0^{\mathrm{C}}}^{\approx 3\%} \leq V_0^{\mathrm{D}} \leq \inf_r \overline{V}_0[r]$$

$$\sup_p \underline{V}_0[p] \leq \overbrace{V_0^{\mathrm{C}} \leq V_0^{\mathrm{D}} \leq \inf_r \overline{V}_0[r]}^{\approx 18\%}$$

- But what can we say about

$$\sup_p \underline{V}_0[p] \leq V_0^{\mathrm{C}} \leq \underbrace{V_0^{\mathrm{D}} \leq \inf_r \overline{V}_0[r]}_{\textbf{Value of the gap?}}$$

$$\sup_p \underline{V}_0[p] \leq \underbrace{V_0^{\mathrm{C}} \leq V_0^{\mathrm{D}}}_{\textbf{Information gap?}} \leq \inf_r \overline{V}_0[r]$$

Tools for mixing spatial and temporal decomposition methods
- Upper and lower bounds using spatial decomposition
- Temporal decomposition using dynamic programming
- The case of deterministic coordination processes

Application to the management of urban microgrids
- Nodal decomposition of a network optimization problem
- Numerical results on urban microgrids of increasing size

Another point of view: decentralized information structure
- Centralized versus decentralized information structures
- Bounds for the centralized and decentralized information structures
- Information gap

Conclusion

Analysis of the decentralized case

For the shake of brevity, we introduce the following notation

$$\mathcal{F}_t^i = \sigma(\boldsymbol{W}_0^i, \dots, \boldsymbol{W}_t^i)$$

Consider the constraints that have to be met at node i in the case of a decentralized information structure

$$\boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) \qquad \text{(nodal dynamic constraints)}$$
$$\Delta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) = \boldsymbol{F}_t^i \qquad \text{(nodal balance equation)}$$
$$\sigma(\boldsymbol{U}_t^i) \subset \mathcal{F}_{t+1}^i \qquad \text{(information structure)}$$

By construction, the state $\boldsymbol{X}_t^i$ is a $\mathcal{F}_t^i$-measurable random variable

Thanks to both the nodal balance equation and the information structure, we deduce that the node flow $\boldsymbol{F}_t^i$ is measurable w.r.t. the σ-field $\mathcal{F}_{t+1}^i$

Analysis of the decentralized case

Suppose that $(\boldsymbol{W}^1, \cdots, \boldsymbol{W}^N)$ are independent random processes
Otherwise stated, we add an **independence assumption in space**

At time t, consider now the global coupling constraints $A\boldsymbol{Q}_t + \boldsymbol{F}_t = 0$.
Summing these constraints leads to the aggregate coupling constraint

$$\sum_{i \in \mathcal{V}} \boldsymbol{F}_t^i = 0$$

From the aggregate constraint and the independence assumption,
we deduce that the random variables $\boldsymbol{F}_t$ (and hence $\boldsymbol{Q}_t$)
are **deterministic variables**

Analysis of the decentralized case

According to this conclusion, under the space independence assumption, in case of a decentralized information structure, the global minimisation problem depends on deterministic node flows f and edge flows q variables

$$\begin{aligned} V_0^{\mathrm{D}} &= \min_{f,q} \left(\sum_{i \in \mathcal{V}} J_{\mathcal{V}}^i(f^i) + J_{\mathcal{E}}(q) \right) \quad \text{s.t.} \quad Aq + f = 0 \\ &= \inf_r \left(\sum_{i \in \mathcal{V}} \left(\min_{f_i} \; J_{\mathcal{V}}^i(f^i) \; \text{ s.t. } \; f^i = -(Ar)^i \right) \; + \; \left(\min_q \; J_{\mathcal{E}}(q) \; \text{ s.t. } \; q = r \right) \right) \\ &= \inf_r \overline{V}_0[r] \end{aligned}$$

The upper bound $\inf_r \overline{V}_0[r]$ and the optimal value V_0^{D} are **the same**

$$V_0^{\mathrm{D}} = \inf_r \overline{V}_0[r]$$

The information gap is high

Recall the sequence of inequalities relating optimal values and bounds

$$\sup_p \underline{V}_0[p] \;\leq\; \overbrace{V_0^{\mathrm{C}} \;\leq\; V_0^{\mathrm{D}} \;\leq\; \inf_r \overline{V}_0[r]}^{\approx 18\%}$$

Gathering all the theoretical and numerical results obtained, we get

$$\underbrace{\sup \underline{V}_0[p] \;\leq\; V_0^{\mathrm{C}}}_{\approx 3\%} \;,\; \underbrace{V_0^{\mathrm{C}} \;\leq\; V_0^{\mathrm{D}}}_{\approx 18\%} \;,\; V_0^{\mathrm{D}} \;=\; \inf \overline{V}_0[r]$$

that provides a way to **quantify the information gap** in our application

Conclusions

- We have two algorithms that decompose spatially and temporally a large-scale optimization problem under coupling constraints.
- In our case study, price decomposition beats SDDP for large instances ($\geq$ 24 nodes)
  - in computing time (more than twice faster)
  - in precision (more than 1% better)
- Price decomposition gives (in a surprising way) a tight lower bound, whereas the upper bound given by resource decomposition is weak (which is understandable on the case study)
- We have studied the case of a decentralized information structure to explain this weakness (information gap)
- Can we obtain tighter bounds?
  If we select properly price $\boldsymbol{P}$ and resource $\boldsymbol{R}$ processes among the class of Markovian processes (instead of deterministic ones) we can obtain "better" nodal value functions (with an extended local state)

Further details in

F. Pacaud
Decentralized Optimization Methods for Efficient Energy Management under Stochasticity
PhD Thesis, Université Paris Est, 2018

P. Carpentier, J.-P. Chancelier, M. De Lara and F. Pacaud
Mixed Spatial and Temporal Decompositions for Large-Scale Multistage Stochastic Optimization Problems
Journal of Optimization Theory and Applications,
Volume 186, Number 3, September 2020

F. Pacaud, M. De Lara, J.-P. Chancelier and P. Carpentier
Distributed Multistage Optimization of Large-Scale Microgrids under Stochasticity
IEEE Transactions on Power Systems, accepted for publication, 2021

THANK YOU FOR YOUR ATTENTION