Matching markets:
Recent developments

Paula Jaramillo (Universidad de Los Andes)
and
Çağatay Kayı (Universidad del Rosario)

Quantil
21 November 2019

Recent Developments

- Transparency and matching markets.
- Limited attention and matching markets.
- Dynamic matching markets.
- Learning in matching markets.

Transparency

- Widespread perception that agency relationships should be as transparent as possible.
- On the other hand, it has been suggested that lack of transparency can help deter gaming.
- That would lead to the "*maximization of the inducement afforded to exertion on the part of learners, by impossibilizing the knowledge as to what part the field of exercise the trial will be applied to, and thence making aptitude of equal necessity in relation to every part*."

  (Official Aptitude Maximized; Expense Minimized As shewn in the several papers compromised in the volume by Jeremy Bentham, London, 1830.)

Transparency

- Assessment of performance of medical doctors (Bevan and Hood, 2004).
- Incentive schemes for physicians and hospital (Kessler, McClellan, and Satterthwaite, 2003).
- U.S. News ranking methodology (Osler, 2010).
- Vague taxation rules (Weisbach, 2000) and vague standards in contracts (Scott and Triantis, 2006).
- Google search algorithmhs.

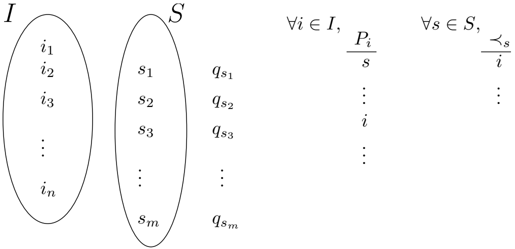
Why are we interested in transparency and in which context?

- Many cities assign students to public schools using rules designed to recommend allocations with desirable properties (Boston, New York City, Chicago, among others).
- In the U.S., the school choice mechanisms used to assign students to public schools are fully transparent whereas in Bogotá, Colombia the school choice mechanism is not transparent at all, i.e., the mechanism is not known by the parents.
- We want to know whether full transparency is “better” than partial transparency or no transparency at all.

Some papers on transparency

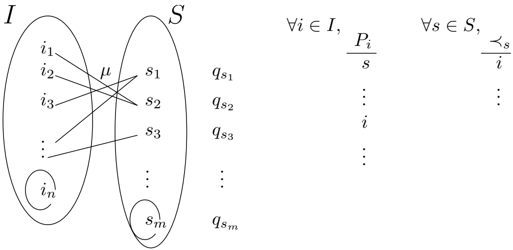
- Morris, S., and Shin, H. S., (2002) "Social value of public information," AER.
- Svensson, L. E. O. (2006) "Social value of public information: Morris and Shin (2002) is actually pro transparency, not con," AER.
- Prat, A. (2005) "The wrong kind of transparency," AER.
- Jehiel, P. (2015) "On transparency in organizations," REStud.
- Brandes, L. and Darai, D. (2017) "The value and motivating mechanism of transparency in organizations," EER.
- Hansen, S. , McMahon M., and Prat, A. (2018) "Transparency and deliberation within the FOMC: A computational linguistics approach," QJE.
- Ederer, F., Holden R., and Meyer M. (2018) "Gaming and strategic opacity in incentive provision," RAND JE.
- Frankel, A. and N. Kartik (2019) "Muddled information," JPE.

School choice problem



- a set of students $I = \{i_1, \ldots, i_n\}$,
- a set of schools $S = \{s_1, \ldots, s_m\}$,
- a capacity vector $q = (q_{s_1}, q_{s_2}, \ldots, q_{s_m})$,
- a list of strict student preferences $P = (P_{i_1}, \ldots, P_{i_n})$ and
- a list of strict school priorities $\prec = (\prec_{s_1}, \ldots, \prec_{s_m})$.
- for each P_i, there exists a vNM utility values $u_i = (u_i(s_1), ..., u_i(s_m), u_i(i))$.
- A problem $(P, \prec, q)$, P for simplicity.

Assignments and rules



- An assignment is a correspondence μ on the set $I \cup S$ such that
  - for each $i \in I$, either $\mu(i) \in S$ or $\mu(i) = i$,
  - for each $s \in S$, either $\mu(s) \subset I$ or $\mu(s) = s$,
  - for each $s \in S$, $|\mu(s)| \leq q_s$, and
  - for each $i \in I$ and $s \in S$, $\mu(i) = s \iff i \in \mu(s)$.
- For each P, let $A(P)$ be the set of assignments.
- A rule is a single-valued function, $\varphi(P) \in A(P)$.

Stability

- μ is non-wasteful if there is no student i and a school s such that
  1. $s \; P_i \; \mu(i)$ and
  2. $|\mu(s)| < q_s$.
- μ is individually rational if for all $i \in I$,

$$\mu(i) \; R_i \; i.$$

- student i has justified envy at μ if there is a school s such that
  1. $s \; P_i \; \mu(i)$ and
  2. $\exists j \in \mu(s)$ such that $i \prec_s j$.

Stability

- μ stable if it is *individually rational*, *non-wasteful*, and no student has *justified envy*.
- Set of stable assignments $\Sigma(P)$.

Transparent game

- Let φ be a rule.
- Each student i reports a preference relation $P_i \in \mathcal{P}$.
- P^* is a Nash equilibrium if for each $i \in N$ and each $P_i' \in \mathcal{P}$,

$$\varphi_i(P_i^*, P_{-i}^*) \; R_i \; \varphi_i(P_i', P_{-i}^*).$$

- $NE(\Gamma(\varphi, P))$ be the set of Nash equilibria of the game.
- $O(NE(\Gamma(\varphi, P))$ be the set of outcomes.

Transparent game

What we knew:

- Ergin and Sönmez (2006): The immediate acceptance rule Nash implements the set of stable matchings, i.e., $O(NE(\Gamma(\varphi^{IA}, P)) = S(P)$.
- Ergin and Sönmez (2006): Each monotonic rank-priority rule Nash implements the set of stable matchings.
- Jaramillo et al. (2019): Each quasi-monotonic rank-priority rule "almost" Nash implements the set of stable matchings.

Transparent game

μ favors priorities at the top if for each $i \in I$, each $s \in S$ such that $r_i(s) = 1$ and $\mu(i) \neq s$, then

1. $|\mu(s)| = q_s$ and
2. for each $j \in \mu(s)$ such that $j \prec_s i$.

Transparent game

What we now know:

- The immediate acceptance with skips rule Nash implements the set of stable matchings.
- If a rule φ selects an *individually rational* assignment that *favors priorities at the top*, then $S(P) \subseteq O(NE(\Gamma(\varphi^{IA}, P))$.

Opaque game

- Let $\alpha \in [0, 1]$ and φ, φ' be two rules.
- Each student i reports a preference relation $P_i \in \mathcal{P}$.
- P^* is a Bayesian–Nash equilibrium if for each $i \in N$ and each $P_i' \in \mathcal{P}$,
$$\alpha u_i(\varphi_i(P_i^*, P_{-i}^*)) + (1-\alpha) u_i(\varphi_i'(P_i^*, P_{-i}^*)$$
$$\geq$$
$$\alpha u_i(\varphi_i(P_i', P_{-i}^*)) + (1-\alpha) u_i(\varphi_i'(P_i', P_{-i}^*)).$$
- $BNE(\Gamma(\alpha, \varphi, \varphi', P))$ be the set of Bayesian–Nash equilibria of the game.
- $O(BNE(\Gamma(\alpha, \varphi, \varphi', P))$ be the set of outcomes.

Opaque game

What we know now:

- $\alpha\varphi^{IA} + (1-\alpha)\varphi^{IAS}$ Bayes–Nash implements the set of stable matchings.
- If φ and φ' select *individually rational* assignments that *favor priorities at the top*, then $S(P) \subseteq O(BNE(\Gamma(\alpha, \varphi, \varphi', P))$.

Where from here:

- Hope to get full implementation results for opaque game.
- Running experiments could be a way to see the effects of transparency over efficiency, stability, and strategy-profness.

Where from here:

Stability

- φ^{IA}
- φ^{IAS}
- $\alpha\varphi^{IA} + (1-\alpha)\varphi^{IAS}$

Efficiency

- φ^{TTC}
- φ^{SD}
- $\alpha\varphi^{TTC} + (1-\alpha)\varphi^{SD}$

Student-optimal stability

- φ^{DA}
- φ^{SDA}
- $\alpha\varphi^{DA} + (1-\alpha)\varphi^{SDA}$

Strategy-proofness

- φ^{TTC}
- φ^{DA}
- $\alpha\varphi^{TTC} + (1-\alpha)\varphi^{DA}$

Limited Attention and School Choice

- Revealed preference is one of the most influential ideas in economics.
  - x is revealed to be preferred to y if and only if x is chosen when y is also available (Samuelson, 1938).
  - Any choice reversal, therefore, observed both empirically and experimentally, is attributed to irrationality since it cannot be expressed as a preference maximization.
  - A decision maker may prefer x to y but she chooses y when x is present simply because she does not realize that x is also available (Hausman, 2008).

Limited Attention and School Choice

- Masatlıoğlu et al. (2012 AER) provide a choice theoretical foundation for maximizing a single preference relation under limited attention.
- They focus on "attention filters":
  - Top N: A decision maker considers only top N alternatives.
  - Top on each criterion: A decision maker has several criteria and considers only the best alternative(s) on each criterion.
  - Most popular category: A decision maker considers alternatives that belong to the most popular "category" in the market.

Limited Attention and School Choice

- Top N: A decision maker considers only top N alternatives.
  - Consider only the three cheapest suppliers in the market.
  - Consider the N-most advertised products in the market.
  - Consider the products that appear on the first page of the web search and/or sponsored links.
  - As a parent, consider the “best” N schools.

Limited Attention and School Choice

- Top on each criterion: A decision maker has several criteria and considers only the best alternative(s) on each criterion.
  - Consider only a job candidate if she is the best in a program. Or consider the top-two job candidates from all first-tier schools and the top candidate from second-tier schools.
  - Consider only the cheapest car, the safest car, and the most fuel-efficient car on the market.
  - As a parent, consider the top schools in academic success, diversification, and internationalization.

Limited Attention and School Choice

- Most popular category: A decision maker considers alternatives that belong to the most popular “category” in the market.
  - There are several bike shops in the city. The decision maker first checks online to find the store offering the largest variety of bikes and goes to that store.
  - As a parent, consider the schools in which the students has a higher priority.

What we want to do

- Consider one specific ”attention filter”.
- Let the agents with limited attention play the preference revelation game and focus on the equilibrium outcomes.
- What are properties of equilibrium outcomes? Stability? Efficiency? Manipulability?

Dynamic Matching Markets

- Many two-sided matching situations involve multiperiod interaction.
  - School choice with multi-child households.
  - Assignment of children to daycares.
  - Teacher assignment.
- Market mechanisms that coordinate agents on "stable" outcomes are known to be more durable (Roth, 2002).
- Therefore, identifying appropriate formulations of stability for these and similar situations has immense practical relevance.
- Traditional cooperative solutions, such as stability or the core, often identify unintuitive outcomes (or are empty) when applied to such markets.

What we want to do

- We want to focus on teacher assignment problem.
- Teachers are assigned to different schools throughout their careers.
- Unfortunately, we do not have “nice” dynamic mechanisms.
- We have problems with stability and strategic issues.
- Can we define some social “desiderata” about teacher’s assignment?

What we want to do

- Kotowski (2019) introduces the perfect $\alpha-$stability.
- An outcome is $\alpha--$stable in period t if (given the elapsed history) there is no coalition of agents who prefer an alternative period-t assignment given all plausible continuations of the market at the proposed alternative.
- A “plausible” continuation must be$\alpha--$stable in period t +1.
- A perfect $\alpha--$stable outcome is ?-stable in every period.
- The solution posits that agents have foresight about the future, but cautiously evaluate possible outcomes.
- A perfect $\alpha--$stable matching exists, even when assignments are inter-temporal complements.
- The perfect $\alpha--$core, a stronger solution, is nonempty under standard regularity conditions, such as history-independence.
- We want to study perfect $\alpha-$stability and other properties in a teacher assignment problem.

Learning in Matching Markets

- In 2014, we changed the mechanism for obligatory social service in Colombia (El año rural de Ministerio de Salud).
- The mechanism is an adaptation of a deferred acceptance mechanism with a tie–breaking.
- After almost 5 years, the new applicants have started to understand how the tie–breaking works (anecdotal evidence from whatsapp groups and online groups).
- How learning affects the properties of the assignment? Is it possible to find non-manipulable tie–breaking?

Thank you