Numerical Weather Prediction and Data Assimilation

Sebastian Quintero Soto

November 18, 2019

Universidad de los Andes - Quantil

Table of contents

1. Numerical Weather Prediction (NWP)
2. Data Assimilation
3. Bogotá Data

Numerical Weather Prediction (NWP)

Numerical Weather Prediction Models

There are several methods to carry out NWP. All of them have different requirements, and produce results at different time scales.

- Global Spectral Model
- Coupled Atmosphere-Ocean General Circulation Model
- Meso-Scale Model
- Local Forecast Model
- Atmospheric Transport Model
- Chemical Transport Model

Global Spectral Model

Let's take a closer look at one of the models, namely, the Global Spectral Model. The model calculates the state of the atmosphere, numerically solving equations of parameterized physical processes. The equations governing the dynamics of the states are:

$$\frac{d\vec{u}}{dt} = \left(\frac{\partial}{\partial t} + \vec{u} \cdot \nabla + \dot{\eta}\frac{\partial}{\partial \eta}\right)\vec{u} \tag{1}$$

$$\frac{dT}{dt} = \frac{\kappa T_V \omega}{\left[1 + \left(C_{pv}/C_{pd} - 1\right) q\right] p} + F_T \tag{2}$$

$$\frac{dq}{dt} = F_q \tag{3}$$

$$\frac{dq_c}{dt} = F_c \tag{4}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial \eta}\right) + \nabla \cdot \left(\vec{u}\frac{\partial p}{\partial \eta}\right) + \frac{\partial}{\partial \eta}\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right) = 0 \tag{5}$$

Finite Differences Method

To solve these equations, the method of finite differences is used. Equations (1) - (5) are written as

$$\frac{d_H X}{dt} = \frac{\partial X}{\partial t} + \vec{u} \cdot \nabla X = R \tag{6}$$

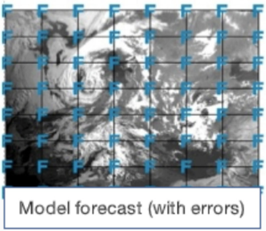
where $X = \{\vec{u}, T, q, q_c\}$ are the prognostic variables.
The discretization comes as follows:

$$X^{A+} - X^{D0} = \Delta t \frac{R^{A0} + R^{D+}}{2} + \Delta t \beta \left[\frac{L^{A+} + L^{D+}}{2} - \frac{L^{A0} + L^{D0}}{2} \right] \tag{7}$$

where $X^{A+} = X(\vec{x} - \vec{\alpha}, t + \Delta t)$ and $X^{D0} = X(\vec{x}, t)$

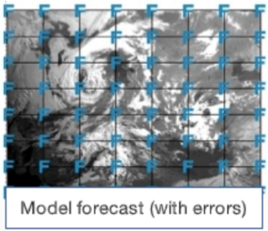
Data Assimilation

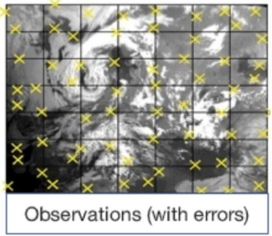
Data Assimilation consist in the reconciliation of the preliminary predictions (Background) and the observed data.



Observation and Background Analysis

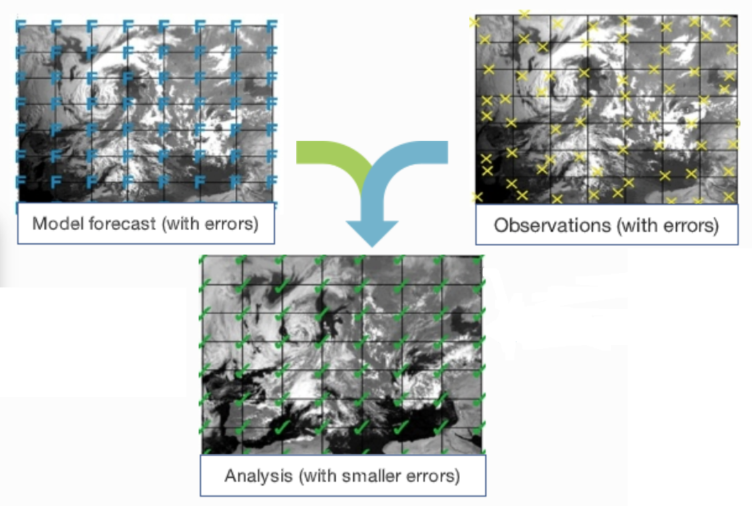
Data Assimilation consist in the reconciliation of the preliminary predictions (Background) and the observed data.





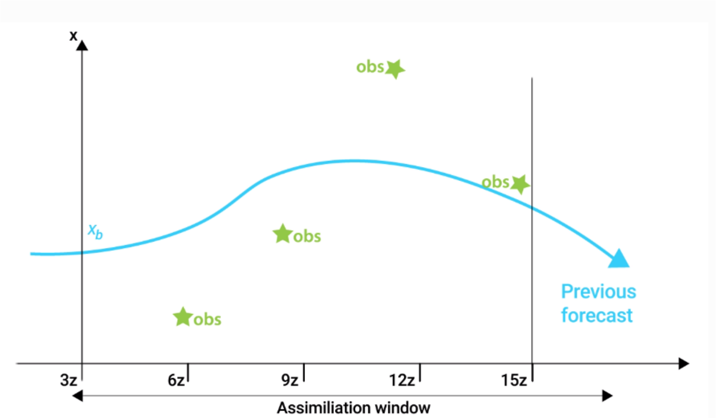
Observation and Background Analysis

Data Assimilation consist in the reconciliation of the preliminary predictions (Background) and the observed data.



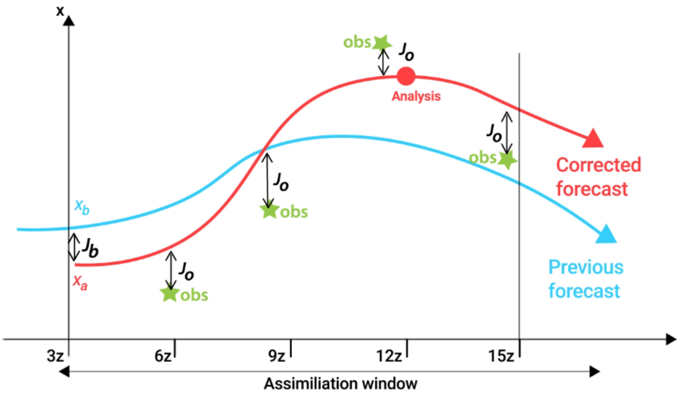
Observation

Let's consider the following assimilation window.



Observation

Let's consider the following assimilation window.



Simple Case

Suppose we have 2 observations Y_1 and Y_2 of the true state X_t.

$$Y_1 = X_t + \epsilon_1 \quad Y_2 = X_t + \epsilon_2 \tag{8}$$

We also have relevant statistical information of the error $\epsilon = (\epsilon_1, \epsilon_2)^T$

$$E[\epsilon] = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad var(\epsilon) = \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} \tag{9}$$

An unbiased estimator for X_t could be

$$\hat{X}_t = \frac{\sigma_2^2 Y_1 + \sigma_1^2 Y_2}{\sigma_2^2 + \sigma_1^2} \tag{10}$$

Cost Function

We could also find the same estimator by minimizing the following function.

$$J(X_a) = \frac{(X_a - Y_1)^2}{\sigma_1^2} + \frac{(X_a - Y_2)^2}{\sigma_2^2} \tag{11}$$

Minimizing J is the same as maximizing $e^{-J/2}$. Thus, we can estimate X_t using maximum likelihood estimation. The extension now to multiple observations is now trivial. For $\vec{Y} = (Y_1, Y_2, \ldots, Y_n)^T$

$$J(X_a) = (X_a - \vec{Y})^T \Sigma (X_a - \vec{Y}) \tag{12}$$

For data assimilation, we want to consider both errors coming from observation and from the background. Therefore, the complete cost function is

$$J(X_a) = J_b(X_a) + J_o(X_a) \tag{13}$$

Bogotá Data

Data

We have meteorological data measures from 13 stations distributed throughout Bogotá.Some of the measured variables are:

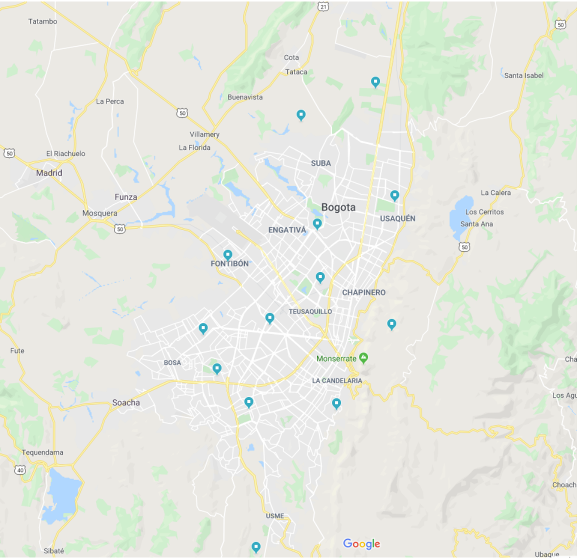
Prognostic variables.

- Temperature
- Wind speed
- Wind direction
- Precipitation
- Solar radiation
- Relative humidity

Chemical variables.

- Particulate matter PM_{10}
- O_3
- NO_2
- SO_2
- CO_2
- CO

Map of Stations



Pollution Threshold

According to the European Commission, the daily limit for the concentration of PM_{10} is 50 $\mu g/m^3$. This limit should not be exceeded more than 35 times in a single year.

Year	Count E1	Count E2	Count E3	Count E4	Count E5	Count E6	Count E8	Count E9	Count E11	Count E13	Count E14
2009	78	105	338	166	85	90	39	320	270	208	215
2010	133	96	336	226	141	147	68	345	231	239	240
2011	47	39	323	189	87	94	36	282	184	214	167
2012	67	0	302	149	74	142	37	265	180	146	132
2013	74	120	327	102	53	65	152	304	216	143	132
2014	63	116	307	114	86	46	36	261	148	233	0

References

- Introduction to data assimilation, European Centre for Medium-Range Weather Forecasts. ecmwf.int/assets/elearning/da/da1/story_html5.html
- Japan Meteorological agency, Numerical Weather Prediction Models jma.go.jp/jma/jma-eng/jma-center/nwp/outline2013-nwp/pdf/outline2013_03.pdf
- Jeff Kepert, 4-D Var for dummies, Center for Australian Weather and Climate Research gmao.gsfc.nasa.gov/events/adjoint_workshop-8/present/Sunday/Kepert1.pdf