The Holographic Principle

A rough historical introduction.

The Holographic Principle

Traditional Hologram

Information about 3D object is saved in 2D plate.

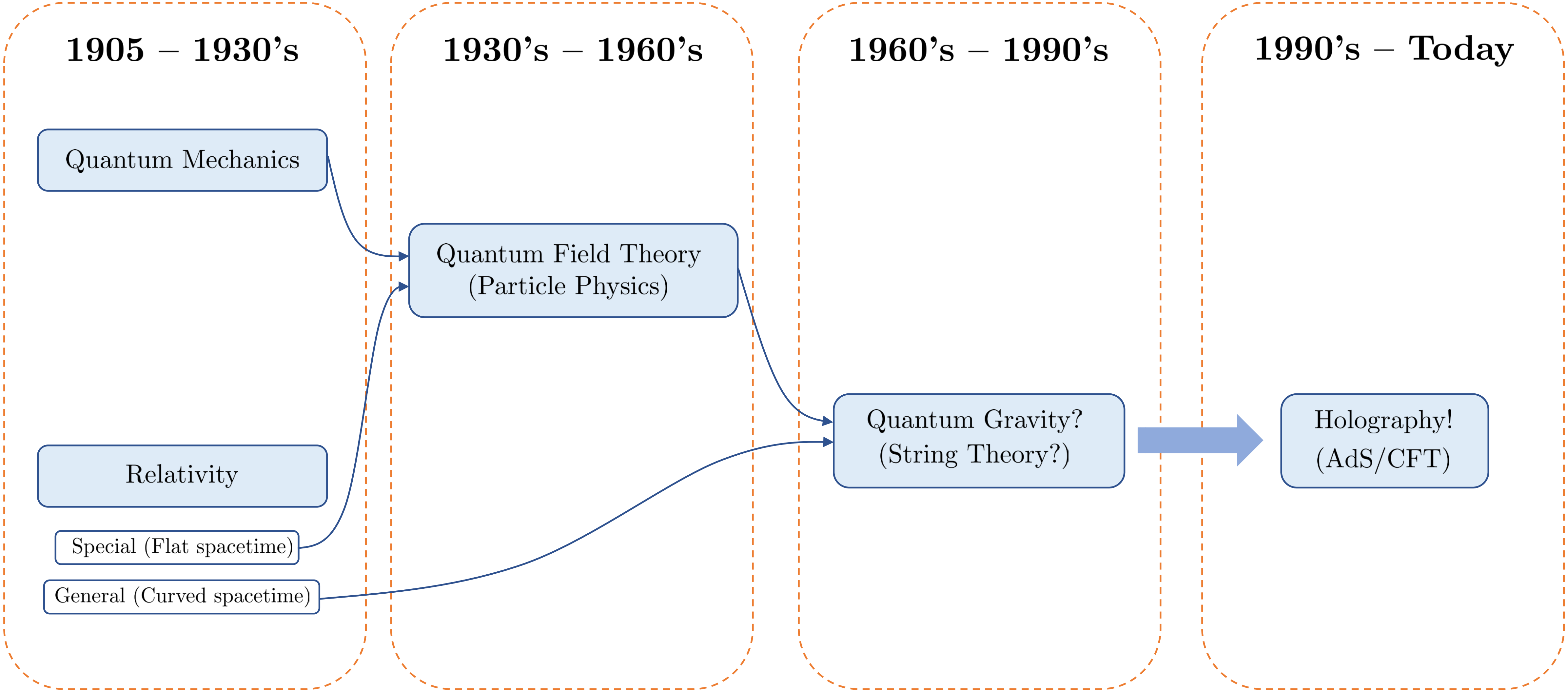
Holographic Universe

We live in a hologram.

Quantum Gravity is highly redundant.

All information about 4D spacetime is stored in a 3D hypersurface.

Overview



1905 – 1930's

Quantum Mechanics

Relativity

Special (Flat spacetime)

General (Curved spacetime)

1930's – 1960's

Quantum Field Theory
(Particle Physics)

1960's – 1990's

Quantum Gravity?
(String Theory?)

1990's – Today

Holography!
(AdS/CFT)

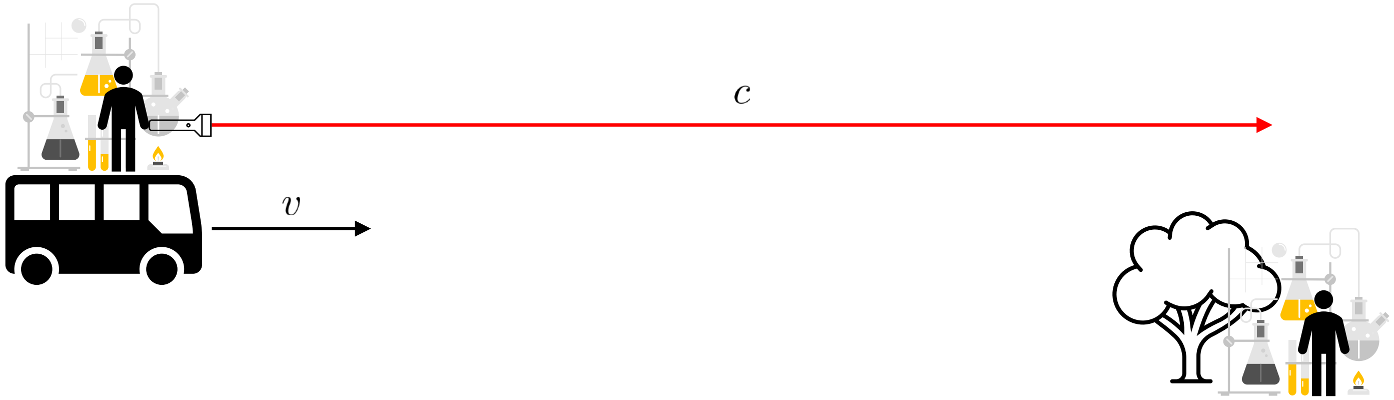
1905 – 1930’s

Quantum Mechanics and Relativity

Special Relativity

Essence: The speed of light c is constant for *all* inertial obsevers

[Einstein 1905]



Both measure $c \approx 3 \times 10^5 \frac{m}{s}$!

Consequences of SR

Most famous consequence

Matter and energy are interchangeable

$$\boxed{E = mc^2}$$

Time Dilation and Length Contraction

From the point of view of a resting observer
the moving observer experiences longer time and shorter distances.

$$\Delta t' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\Delta t$$

$$\Delta x' = \sqrt{1-\frac{v^2}{c^2}}\Delta x$$

Trip to Proxima Centauri (4.2 ly) at 0.8c:

	Earth	Spaceship
Travel distance:	4.2ly	$\sqrt{1-0.8^2}4.2 \text{ ly} = 2.52\text{ly}$
Travel time:	$\frac{4.2}{0.8}y = 5.25y$	$\frac{2.52}{0.8}y = 3.15y$

General Relativity

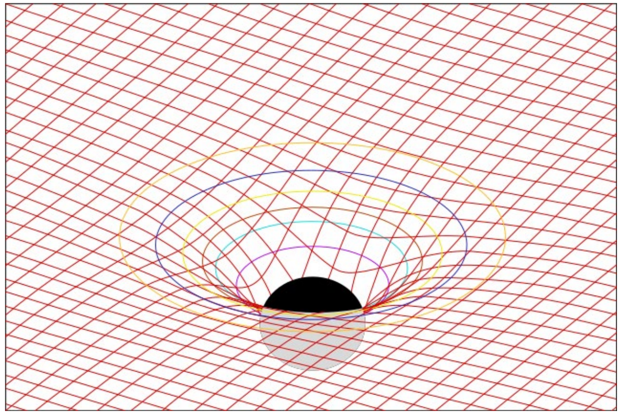
Essence: Spacetime is a curved manifold and gravity = curvature

[Einstein 1915]

Einstein equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$



Consequences of GR

Time Dilation and Length Contraction

A clock in orbit runs faster than a clock on earth.
GPS has to take this into account!

Black holes

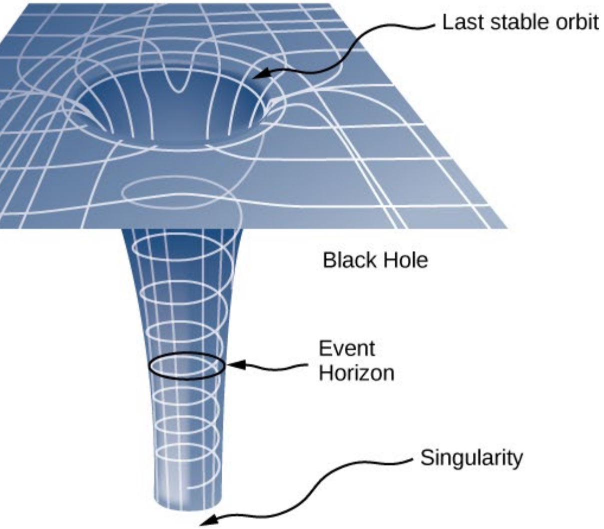
[Schwarzschild 1916]

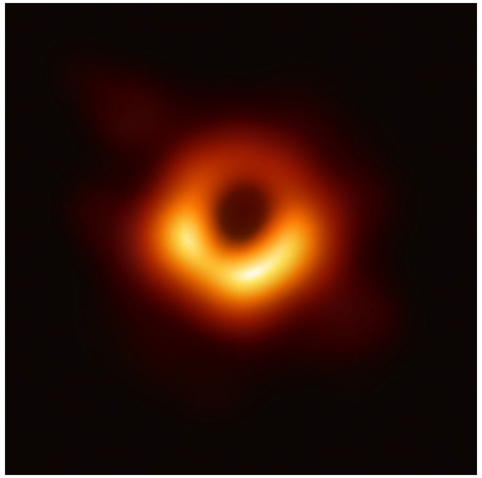
Enough energy/mass in small enough region of spacetime

$\Rightarrow$

Spacetime region collapses into singularity

Event Horizon:
Boundary of region from which nothing can escape.




[Event Horizon Telescope 2017]

Quantum Mechanics

Essence: At the nanoscopic level things are quantized.
[Planck 1900]

The Photoelectric Effect:
[Einstein 1905]

Shining light on a metal plate causes electrons to be emited.

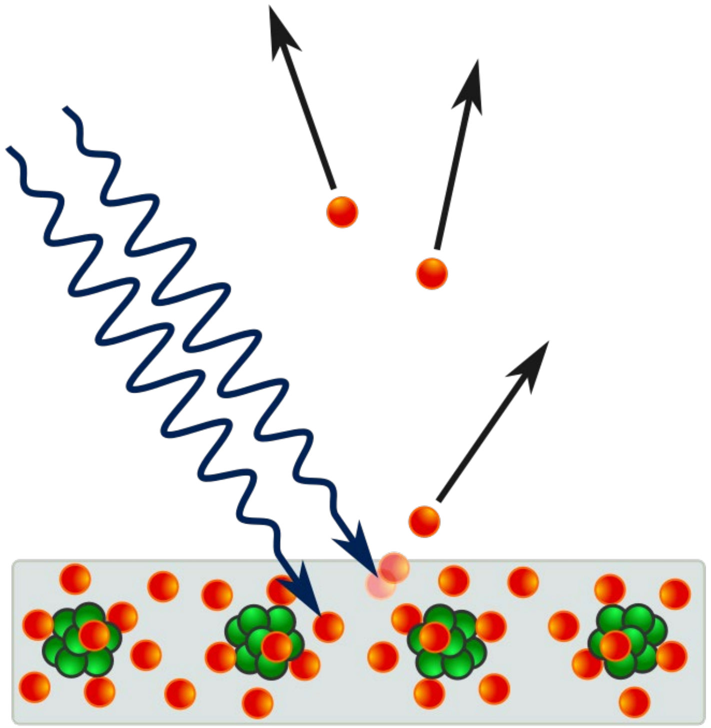
Classical: Kinetic energy depends on intensity of light.

Quantum: Kinetic energy depends on frequency of light.

Photon Energy:

$$E = \hbar\nu$$

$$\hbar = 1.055 \times 10^{-34} Js$$



Mathematical Formulation of QM

Postulate I:

The state of a quantum system is described by
a unit vector $|\psi\rangle$ in a Hilbert space $\mathcal{H}$

Postulate II:

An observable A is described by a Hermitian operator on $\mathcal{H}$.
A measurement projects $|\psi\rangle$ onto an eigenvector of A.
We observe the corresponding eigenvalue.

Postulate III:

Time evolution is goverened by the Schrödinger equation

$$i\hbar\partial_t|\psi(t)\rangle = H(t)|\psi(t)\rangle$$

[Heisenberg, Schrödinger, Dirac, Born, von Neumann, Weyl et al 1930's]

Consequences of QM

Uncertainty

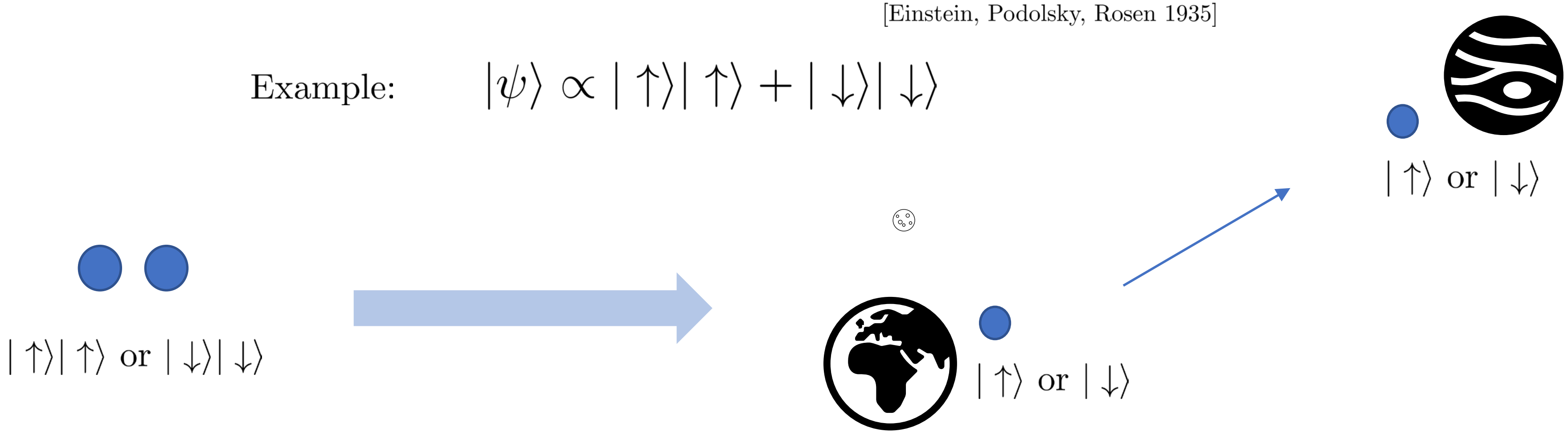
Measurement of non-commuting Observables has inherent uncertainty.
[Heisenberg 1927]

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Entanglement

∃ systems consisting of multiple parts that can only be treated as a whole.
[Einstein, Podolsky, Rosen 1935]

Example: $|\psi\rangle \propto |\uparrow\rangle|\uparrow\rangle + |\downarrow\rangle|\downarrow\rangle$



$|\uparrow\rangle|\uparrow\rangle$ or $|\downarrow\rangle|\downarrow\rangle$

$|\uparrow\rangle$ or $|\downarrow\rangle$

$|\uparrow\rangle$ or $|\downarrow\rangle$

1930's – 1960's

Quantum Field Theory

Motivation for QFT

From Quantum Mechanics

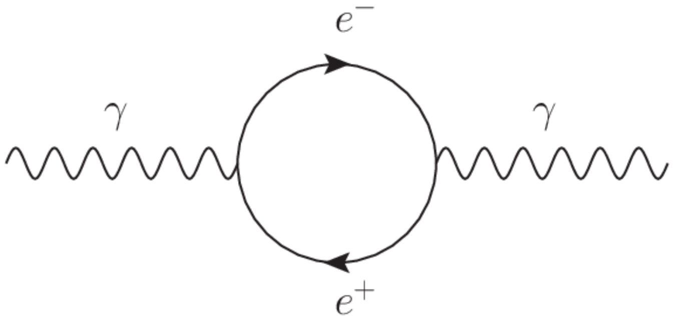
$$\Delta E \Delta t \geq \hbar$$

Consider a sufficiently small time interval,
then the energy uncertainty permits arbitrarily high energies

From Special Relativity

$$E = mc^2$$

Sufficient energy can be transformed into massive particles



Particles can always be temporarily produced:

Does it even make sense to talk about single particle processes?

Quantum Field Theory

Essence: Particles are the excitations of quantum fields.

Quantum Electrodynamics (QED)

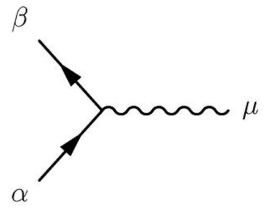
$$\mathcal{L} = \boxed{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}} + \boxed{\bar{\psi}i\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi} - \boxed{e\bar{\psi}\gamma^{\mu}\psi A_{\mu}}$$

[Feynman, Tomonaga, Schwinger, Dyson 1940's]

Electron Field: $\psi, \bar{\psi}$

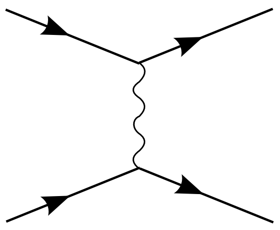
Photon Field: $F_{\mu\nu} \equiv \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$

Coupling:

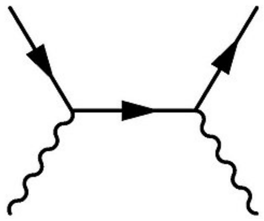


Energy can be freely exchanged between electron and photon fields!

Feynman Diagrams:



Coulomb potential



Compton Scattering

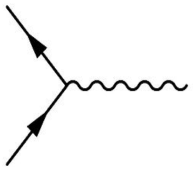
Perturbative QFT:

The anomalous magnetic moment of e^-

Calculate a process = Sum all Corresponding Feynman diagrams

Magnetic moment of e^-

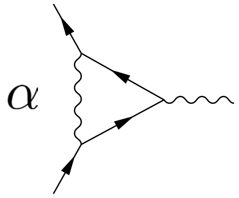
Electron behaves like small magnet with strength determined by g.
QM predicts $g = 2$, process corresponds to



Anomalous MM of e^-

[Schwinger 1948]

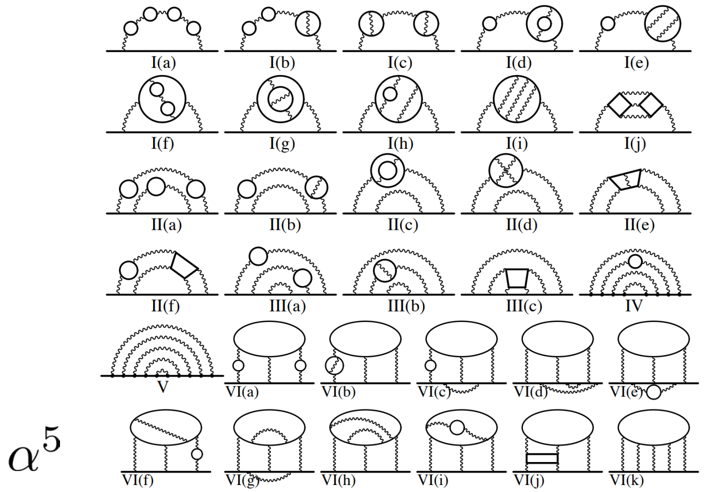
Add higher order Feynman diagrams to gain precision.
Each vertex proportional to $\sqrt{\alpha} \approx \frac{1}{\sqrt{137}}$



$$\text{Prediction } (\alpha^5): \quad a = \frac{g-2}{2} = 0.001\,159\,652\,181\,643(764)$$

$$\text{Experiment:} \quad a = 0.001\,159\,652\,180\,73(28)$$

[Aoyama et al. 2020]



Entanglement in QFT

Entanglement Entropy

Measures the amount of entanglement between complementary regions

$$S = \mathrm{Tr}\,\rho_A \log \rho_A$$

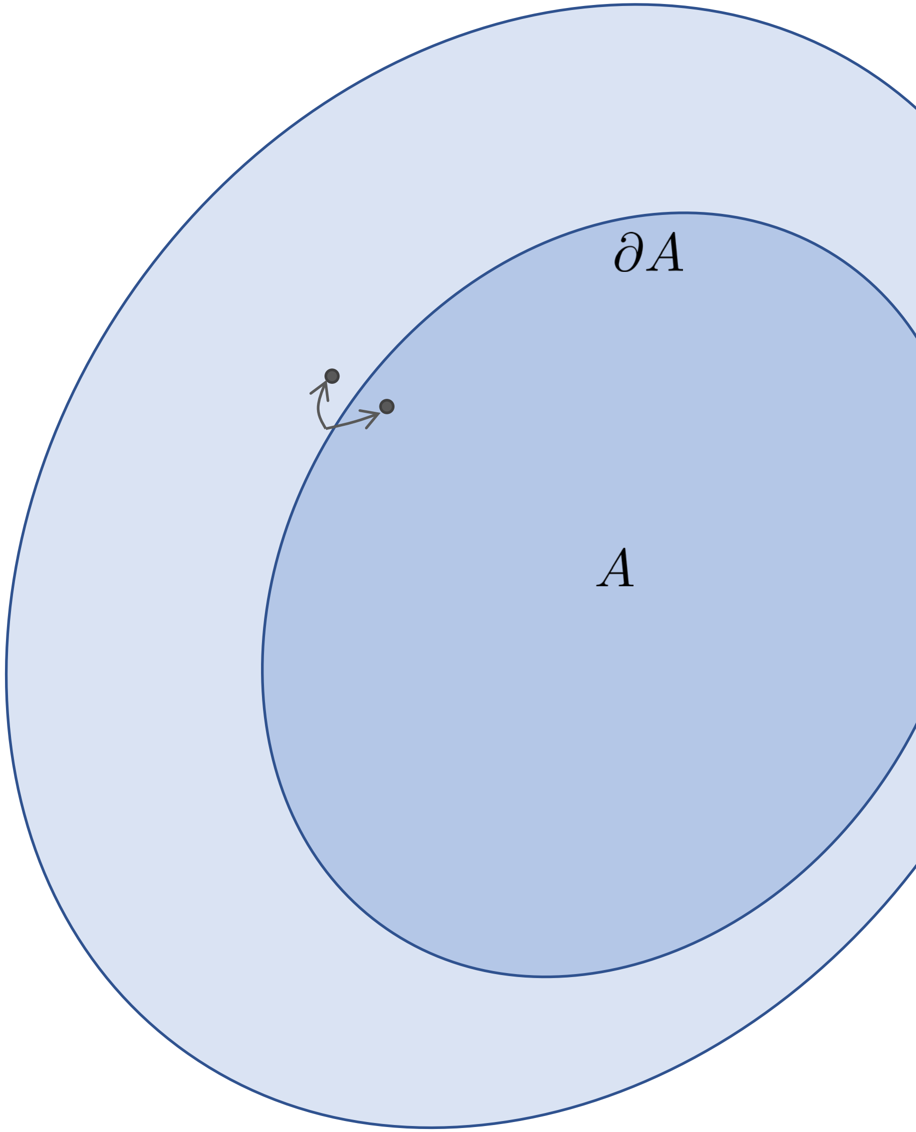
Ground states and Area laws

[Bombelli, Srednicki 1986 and 93]

Entanglement is infinite since we can always generate entangled pairs at ∂A

Leading term is proportional to the area of the boundary ∂A

$$S = c\frac{\mathrm{Area}(\partial A)}{\varepsilon} + \mathcal{O}(\varepsilon^0)$$



The Standard Model

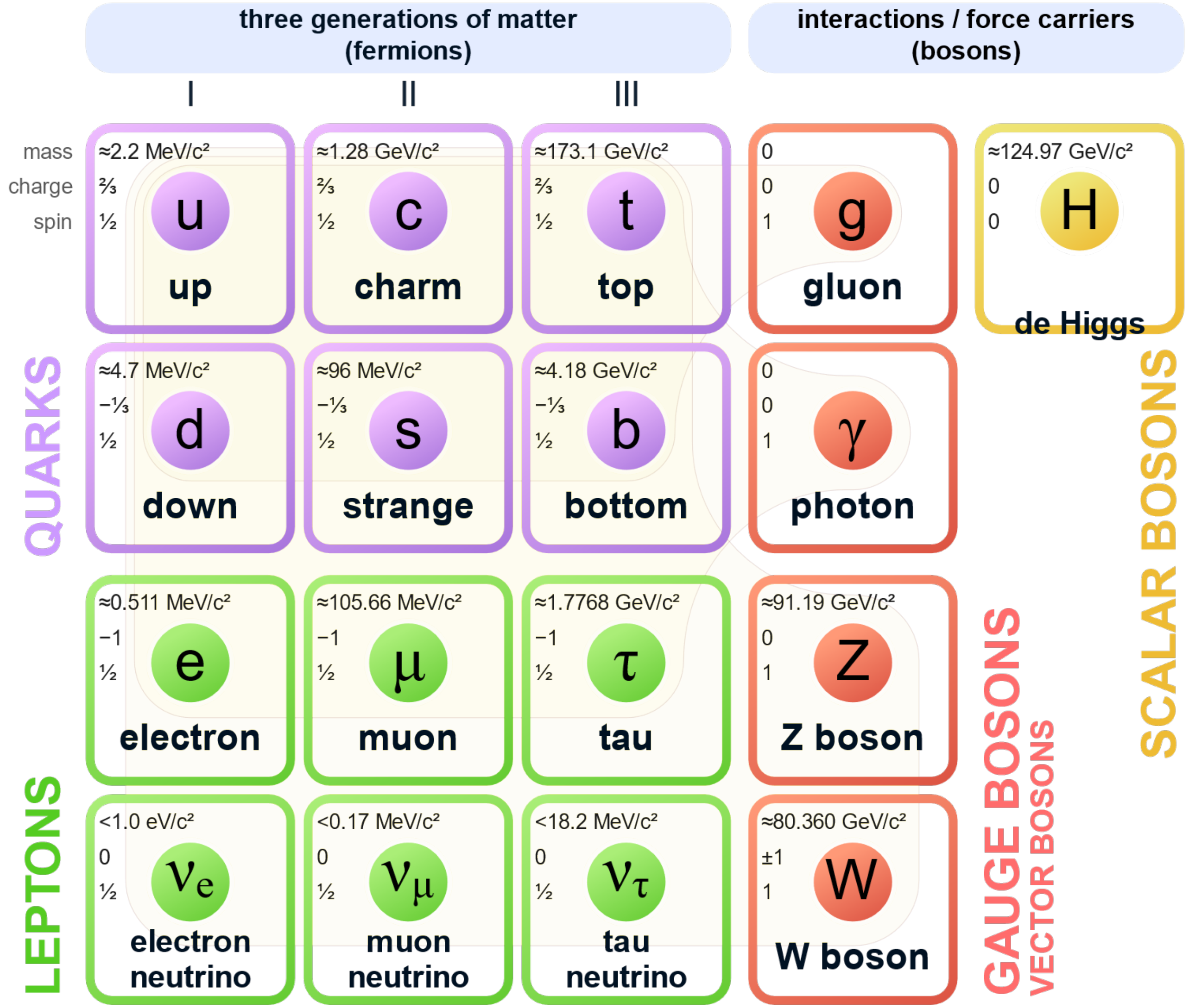
Three Fundamental Forces

Electromagnetism
Quantum Electrodynamics

Weak Nuclear Force
with QED: Electroweak Theory

Strong Nuclear Force
Quantum Chromodynamics

What about Gravity?
Naive QFT breaks down



	three generations of matter (fermions) I	II	III	interactions / force carriers (bosons)	
mass charge spin					
QUARKS	≈2.2 MeV/c^2 ⅔ ½ u up	≈1.28 GeV/c^2 ⅔ ½ c charm	≈173.1 GeV/c^2 ⅔ ½ t top	0 0 1 g gluon	≈124.97 GeV/c^2 0 0 H de Higgs
QUARKS	≈4.7 MeV/c^2 −⅓ ½ d down	≈96 MeV/c^2 −⅓ ½ s strange	≈4.18 GeV/c^2 −⅓ ½ b bottom	0 0 1 γ photon	SCALAR BOSONS
LEPTONS	≈0.511 MeV/c^2 −1 ½ e electron	≈105.66 MeV/c^2 −1 ½ μ muon	≈1.7768 GeV/c^2 −1 ½ τ tau	≈91.19 GeV/c^2 0 1 Z Z boson	GAUGE BOSONS VECTOR BOSONS
LEPTONS	<1.0 eV/c^2 0 ½ ν_e electron neutrino	<0.17 MeV/c^2 0 ½ ν_μ muon neutrino	<18.2 MeV/c^2 0 ½ ν_τ tau neutrino	≈80.360 GeV/c^2 ±1 1 W W boson	

1960’s – 1990’s

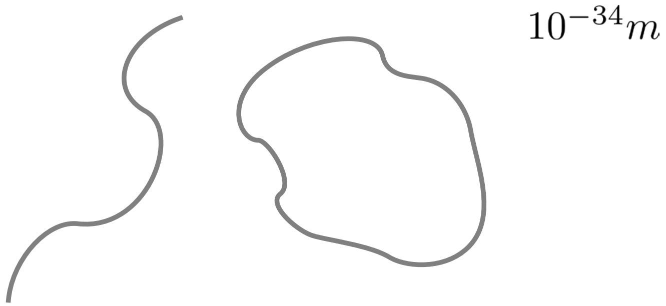
The Quest for Quantum Gravity

String Theory

Essence

At the smallest scales particles are not point-like, but one-dimensional strings.

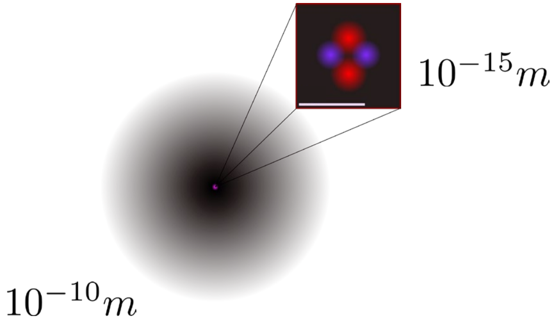
Properties of particles (charge, spin) determined by vibrations of the string.



Why?

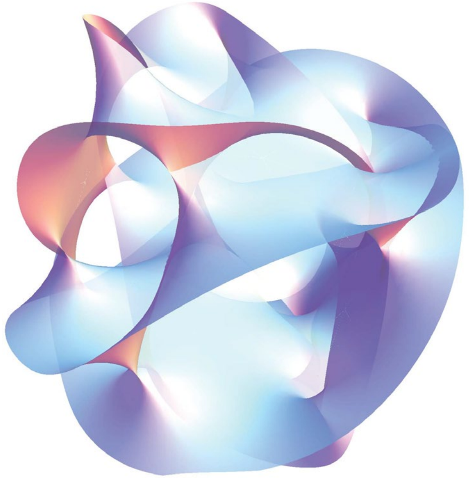
Originally (1960's): Understand internal structure of protons and neutrons.

In the 1980's: String theories describe all forces!



String Theory

Some Caveats



Extra Dimensions

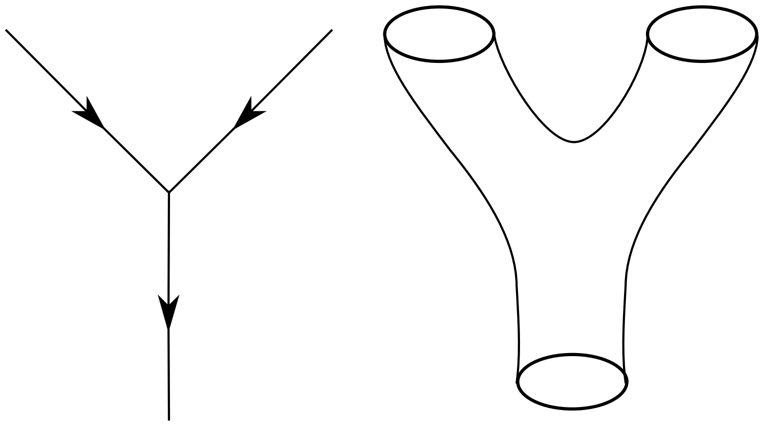
Superstring Theories require 10 dimensions.
Reconciling this with 4D spacetime requires compactification.

Experimental Evidence

Incredibly small scale requires incredibly large energies.
So far no direct evidence.

Complicated

Highly non-trivial to do almost anything in string theory.



Particles and Black Holes

What happens if we put a QFT on a curved background?

Hawking Radiation

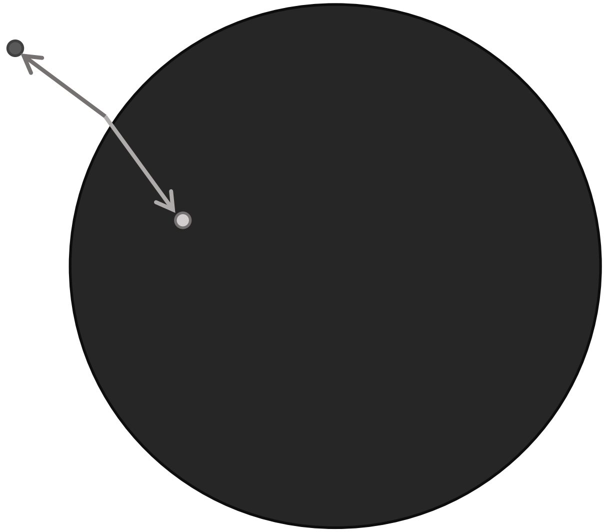
$T_H \propto \frac{1}{M}$

[Bekenstein, Hawking 1970's]

Pair production at the event horizon:

Sometimes one of the particles escapes, while the other falls in.

Black hole slowly emits radiation and evaporates.



Bekenstein-Hawking Entropy

$S_{BH} \propto A_{\mathrm{EH}}$

Entropy of a black hole is proportional to the area of its event horizon.

But where does the entropy come from?

String theory can explain the microstates that lead to the area law!

1990's – Today

Holography!

An Epiphany

['t Hooft, Susskind 1990's]

Information Paradox

For a far away observer, any information that enters a black hole is destroyed.
Unitarity in quantum mechanics prohibits destruction of information.

Holographic resolution

Information is stored at the event horizon and emited as Hawking radiation.

→

Anything that can fall into a black hole can be described on its surface.

How?

QFT Entanglement entropy and BH entropy suggest:

Geometry $\Longleftrightarrow$ Quantum Information

The AdS/CFT Correspondence

AKA the Maldacena Conjecture

[Maldacena 1997]

First explicit realization of the holographic principle:

Type IIb Superstring theory on $\mathrm{AdS}_5 \times S^5$

$$\Longleftrightarrow$$

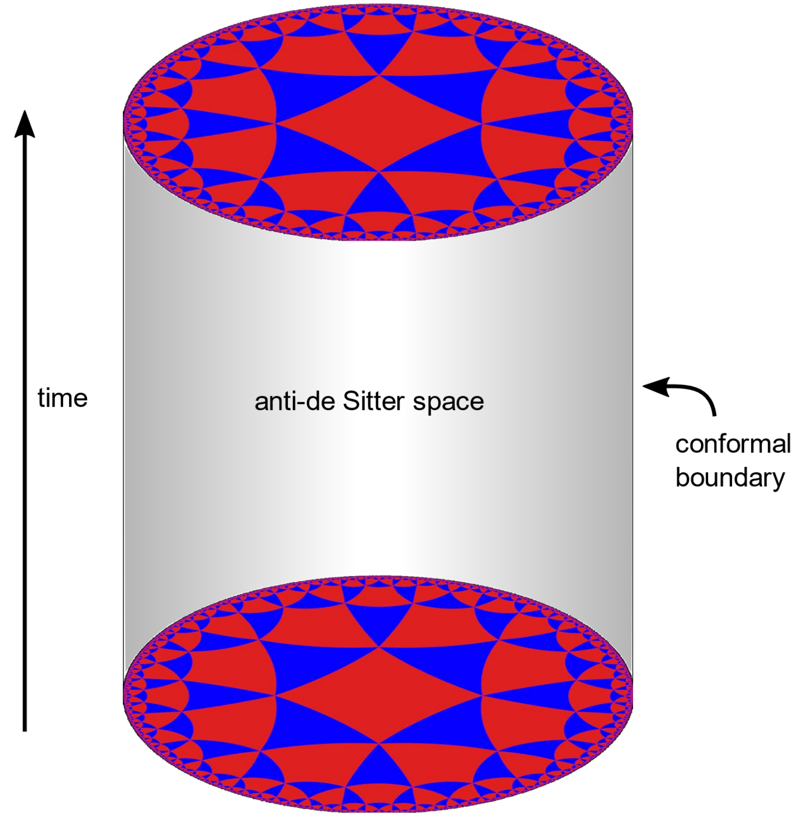
$\mathcal{N} = 4$ Super Yang-Mills on AdS boundary

More generally:

String theory on an AdS background

$$\Longleftrightarrow$$

Conformal Field Theory

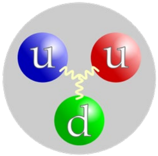


A weak-strong duality

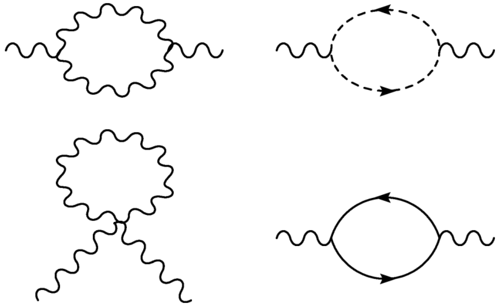
AdS/CFT relates the easy parts of one theory to the hard parts of the other.

QCD
a tentative example

Strongly coupled at low energies, perturbative methods don't converge



We understand the inner structure of protons at the LHC,
We don't understand the inner structure of resting protons.



Holography:

Strongly Coupled QCD in 4D

$$\Longleftrightarrow$$

Classical gravity in 5D

Widespread approach in certain areas of condensed matter physics.

Entanglement and Spacetime

[Ryu and Takayanagi 2006]

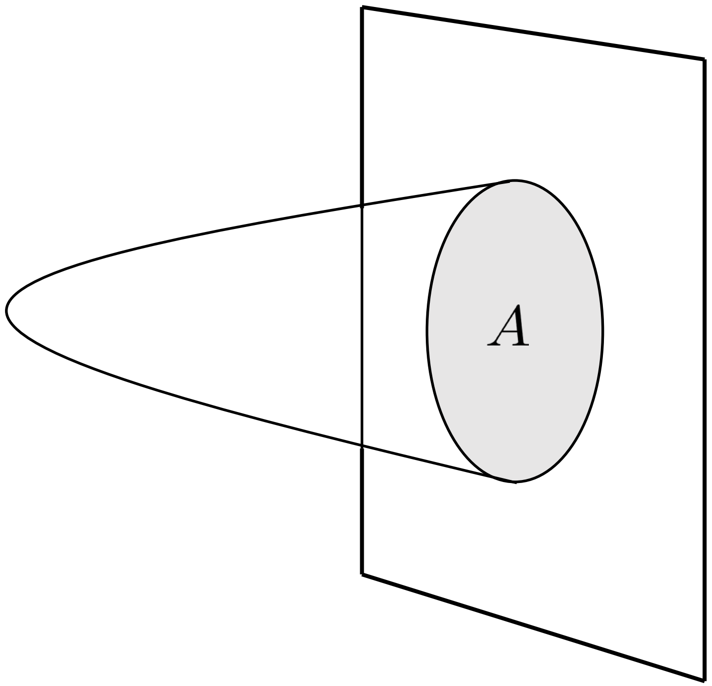
Remember analogy between black hole entropy and entanglement?

$\Rightarrow$

Entanglement entropy of A in boundary QFT

$$\Longleftrightarrow$$

Area of minimal surface with same boundary as A reaching into bulk



Possible to reconstruct the metric in the bulk from entanglement at boundary.

Spacetime at the Planck Scale

Our notion of spacetime breaks down at the Planck scale
$(10^{-35}m)$

Anything small or energetic enough to probe the Planck scale will collapse into a black hole.

So what is spacetime?

Quantum Gravity/Holography:

Spacetime is not fundamental, it emerges from the dynamics of QG.

At small enough scales we should think of it in terms of the entanglement properties of the holographic dual.

The End

Thank you!

[Images taken from the respective articles in Wikipedia]